Stochastic differential equations for the electromagnetic field scattered by the sea surface for remote sensing applications

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Abstract:

When the sea surface is illuminated by a radar sufficiently far away and in absence of a coherent target, the returned electromagnetic signal, called 'sea clutter' is unpredictable and is therefore represented by a stochastic process. A model based on stochastic differential equations (SDE) and consistent with previous statistical models (K distribution) has been proposed for the sea clutter [1]. The complex reflectivity of the sea-surface (sea clutter), Ψ_t, can be expressed as:

$$\Psi_t = x_t^{1/2}(\gamma_t^{(R)} + i\gamma_t^{(I)})$$

where x_t (real-valued process) is the normalized radar-cross section (RCS) and $\gamma_t^{(R)} + i\gamma_t^{(I)}$ (complex-valued process) is the speckle. Field's model [1] states that:

$$\begin{cases} \mathrm{d}x_t = \mathcal{A}(1-x_t)\mathrm{d}t + \left(2\frac{\mathcal{A}}{\alpha}x_t\right)^{\frac{1}{2}}\mathrm{d}W_t^{(x)} \\ \mathrm{d}\gamma_t^{(R)} = -\frac{1}{2}\mathcal{B}\gamma_t^{(R)}\mathrm{d}t + \frac{1}{\sqrt{2}}\mathcal{B}^{\frac{1}{2}}\mathrm{d}W_t^{(R)} \\ \mathrm{d}\gamma_t^{(I)} = -\frac{1}{2}\mathcal{B}\gamma_t^{(I)}\mathrm{d}t + \frac{1}{\sqrt{2}}\mathcal{B}^{\frac{1}{2}}\mathrm{d}W_t^{(I)} \end{cases}$$

where $W_t^{(x)}, W_t^{(R)}, W_t^{(I)}$ are 3 independent brownian motions.

In this article, we address two problems relevant to remote sensing applications.

When a moving sensor (*e.g.* satellite radar imaging) observes the sea, emerges the problem of comparing subsequent measures since the sea clutter has significantly evolved during the sampling interval. Thus, we address the issue of transporting measures to a common time reference using Field's model . We solve the Fokker-Planck equations of the speckle and radar cross-section (RCS) to obtain their present to future transition probabilities, from which we derive those of the intensity $z_t = |\Psi_t|^2$ and the real and imaginary parts of the reflectivity. Using Bayes's formula we extend the results to present to past transition probabilities. Numerical distributions are systematically computed and match the analytical distributions. A series of deterministic measures from different positions and times is transformed into a series of probabilistic measures from different positions at the same time. (see [2] for complementary details)

The used model depends on 3 parameters $\mathcal{A}, \mathcal{B}, \alpha$ to be estimated for applications. We numerically simulate trajectories of the SDE with known parameters, and prove that we can retrieve them using maximum likelihood estimators for $\mathcal{A}$ and $\mathcal{B}$, and an hypothesis of ergodicity for α. We compare 3 expressions for the transition probabilities: the exact one, Euler's approximation, and Nowman's approximation. We propose a criteria for an admissible method of estimation in relation with real data measurement standard deviation. Though the exact transition probabilities minimize the error on the sea clutter, approximations like Euler's or Nowman's are also satisfying. (see [3] for complementary details)

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References

[1] T. R. Field, Electromagnetic Scattering from Random Media. *Oxford University Press*, 2009.

[2] C. J. Roussel, A. Coatanhay and A. Baussard. Forward and backward probabilistic inference of the sea clutter, to be published in *Waves in Random and Complex Media.*

[3] C. J. Roussel, A. Coatanhay and A. Baussard. Estimation of the parameters of stochastic differential equations for the sea clutter, submitted to *Stochastic Analysis and Applications.*