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Identification of the electromagnetic scattering by dynamic sea surfaces with a stochastic differential equation model

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Abstract

This paper presents a Nonlinear Stochastic Differential Equation System (NLSDES) that can be used to model the electromagnetic scattering by time-varying sea surfaces. More precisely, we show how to identify the parameters of this generic stochastic model with the numerical simulations computed for different sea states.

1. Introduction

For remote sensing activities in maritime environment, the scattering of the radar electromagnetic waves by the sea surface is of a major importance to assess the weather conditions (wind speed and wind direction) and the sea dynamics (swell for instance). Due to complex non-linear fluid dynamics, sea surfaces can be considered as random rough surfaces. To model the electromagnetic field scattered by such boundaries, various theoretical approaches were developed: asymptotic approximations (two scale method, small slope approximation) [1, 2], Monte-Carlo simulations based upon numerical techniques, statistic models [3],...

However, most of the time, these models assume static sea surfaces. If a time variation is taken into account, the modeling issue becomes far more challenging. The most obvious way (Monte-Carlo approach) is to generate a great number deterministic sea surfaces, simulate the evolution in time using fluid mechanics and finally compute the scattered field using numerical methods (Method of Moments for instance). Unfortunately, this approach does not provide an efficient analytical model.

More recently, Field and al. developed a Stochastic Differential Equation (SDE) model that can be used for this purpose. Nevertheless, this model requires parameters that have to be identified with actual scattered fields. This papers presents the identification obtained from numerical simulations for different sea states.

2. Numerical simulations

2.1. Sea surface generation

To generate a realistic ocean surface associated to a given weather condition (wind speed and wind direction), we introduce the sea spectrum developed by Elfouhaily et al. [4] since it is very consistent with experimental data. This sea spectrum is in the form:

$$S(K, \phi) = M(K) f(K, \phi) \tag{1}$$

where $M(K)$ represents the isotropic part of the spectrum modulated by the angular function $f(K, \phi)$, and where K and ϕ are respectively the spatial wave number and the wind direction, see figure (1). Then, the convolution of this spectrum with an unitary white Gaussian random signal generates a one-dimensional profile (a statistical realization for the sea surface) that represents an ocean surface for given weather conditions (see fig. 2).

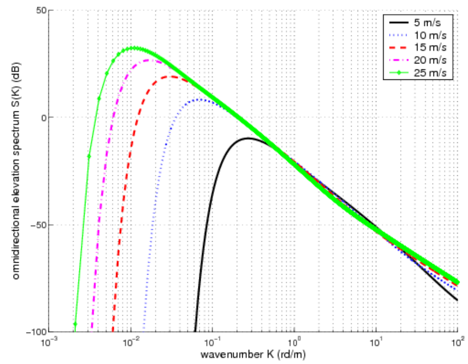

Figure 1: Elfouhaily sea surface spectra with different wind speeds (isotropic component).

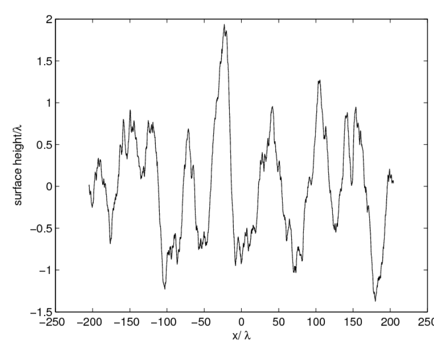

Figure 2: Example of an ocean surface profile generated where the wind speed is $10\,m/s$.

To introduce the movement of the so-generated random profiles, we must take into account the velocity of the sea waves that depends on the wave number [5]. The velocity of the longer wavelength waves is mainly influenced by the gravity, whereas for the shorter wavelength waves the predominant effect in the velocity is the capillary. For non-shallow water, the dispersion relation can be approximated by:

$$\omega\left(k\right) \approx k \cdot \sqrt{\frac{g}{k} + \frac{\tau k}{\rho}} \qquad (2)$$

where g is the gravitational acceleration, τ is the water surface tension and ρ is the density.

2.2. Electromagnetic scattering simulations

To compute the electromagnetic field scattered by the ocean-like profiles previously generated, we apply an accelerated Method of Moments (MoM). Then, using the so-computed scattered fields, we can numerically simulate the temporal stochastic process corresponding to the RCS for different weather conditions. In this way, the numerical data generated by a Monte-Carlo methodology can be seen as the realizations of the stochastic process related to the electromagnetic scattered field.

3. Stochastic differential equation model

Assuming that the sea surface is modeled by a set of random scatterers (phase screens), Field and al.[6, 7] proved that the dynamics of the normalized amplitude process $\Psi_t = \mathcal{E}_t^{(N)}/N^{1/2}$ and the continuous valued RCS x_t are given by the following set of nonlinearly coupled stochastic differential equations (NLSDES):

$$dx_t = \mathcal{A}\left(\alpha - x_t\right)dt + \left(2\,\mathcal{A}x_t\right)^{1/2}dW_t \qquad (3a)$$

$$\frac{d\Psi_t}{\Psi_t} = \left[\mathcal{A}\left(\frac{2\left(\alpha - x_t\right) - 1}{4x_t}\right) - \frac{1}{2}\mathcal{B}\right]dt + \left(\frac{\mathcal{A}}{2x_t}\right)^{1/2} dW_t^{(x)} + \frac{\mathcal{B}^{1/2}}{\gamma_t}d\xi_t \qquad (3b)$$

where $\mathcal{A}$ and $\mathcal{B}$ are constant. It should be noted that W_t and ξ_t are two independent Wiener processes.

Now, the major problem is to identify the both parameters ($\mathcal{A}$ and $\mathcal{B}$) from electromagnetic scattered field previously simulated.

4. Identification of the parameters

The identification of a nonlinear stochastic model is usually a very delicate task, see for instance the recursive approach developed by D. Levanony et al. [8]. In the present study, we developed a more pragmatic approach adapted to the Field and al. stochastic model. The main idea is to decouple the identification of each parameters.

First, the numerical data are based upon a finite difference approximation $dt \approx \Delta t$ and $dx_t \approx x_{t+\Delta t} - x_t$. Based upon the analysis of averaging process for the first equation of the NLSDES, it is possible to provide an accurate estimation of the parameter $\mathcal{A}$. Then, the analytical properties of Wiener process can be used to identify the characteristics of process W_t.

In a second step, following different calculus and mathematical transformations, we can obtain a decoupled identification of the NLSDES model. Moreover, our method also provides an estimated precision for the identified parameters.

5. Conclusion

From the computed data (numerical simulation), our algorithm can identify the parameters of the NLSDES model. Since the simulations are made for various sea states, the parameters of the dynamic stochastic model can be associated to the different sea states (wind speed and wind direction). Finally, our study highlight the relations between parameters $\mathcal{A}$ and $\mathcal{B}$ and the dynamic of the sea in different weather conditions.

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