Analysis of the sea state observability by radar systems: numerical simulations and stochastic diffusion modeling

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Abstract — **This paper shows that connections can be established between numerical simulations and a stochastic differential equation model for the electromagnetic scattering by sea surfaces.**

1 Introduction

The electromagnetic scattering by sea surfaces is a well-known issue abundantly covered in the scientific literature. Many different approaches were developed to model this phenomenon. Oversimplifying, these models are based upon deterministic and asymptotic approaches [1, 2] or upon purely statistical descriptions [3]. Unfortunately, combining these two viewpoints remains an open and complex problem.

Recently, Field and al. [4, 5] developed a Stochastic Differential Equation (SDE) model that can be used for this purpose.

2 Stochastic model

2.1 Diffusion process

A Wiener process W_t is defined [6] as a continuous time stochastic process that verifies:

$$E\left[\left(W_v - W_u\right)\left(W_t - W_s\right)\right] = 0 \tag{1}$$

where $s < t \leq u < v$ and where $W_t - W_s$ is the Gaussian distribution $\mathcal{N}\left(0, |t-s|\right)$. A time continuous stochastic q_t is called a “diffusion” if the differential dq_t is in the form:

$$dq_t = b_t dt + \sigma_t dW_t \tag{2}$$

b_t and σ_t are respectively call the “drift” and the “volatility”.

2.2 Stochastic differential equation model

Let's assume that the sea surface is modeled by a set of random scatterers (phase screens), the electromagnetic scattered wave is the result of the stochastic process:

$$\mathcal{E}_t^{(N)} = \sum_{j=1}^{N} a_j \cdot e^{i\varphi_t^{(j)}} \tag{3}$$

with a fluctuating population size N. Field and al.[4, 5] proved that the dynamics of the normalized amplitude process $\Psi_t = \mathcal{E}_t^{(N)}/N^{1/2}$ and the continuous valued RCS x_t are given by the following set of nonlinearly coupled SDEs:

$$dx_t = \mathcal{A}\left(\alpha - x_t\right)dt + \left(2\,\mathcal{A}x_t\right)^{1/2} dW_t^{(x)} \tag{4a}$$

$$\frac{d\Psi_t}{\Psi_t} = \left[\mathcal{A}\left(\frac{2\left(\alpha - x_t\right) - 1}{4x_t}\right) - \frac{1}{2}\mathcal{B}\right]dt + \left(\frac{\mathcal{A}}{2x_t}\right)^{1/2} dW_t^{(x)} + \frac{\mathcal{B}^{1/2}}{\gamma_t}d\xi_t \tag{4b}$$

where $\mathcal{A}$ and $\mathcal{B}$ are constant. It should be noted that W_t and ξ_t are two independent Wiener processes.

3 Numerical simulations

3.1 Sea surface generation

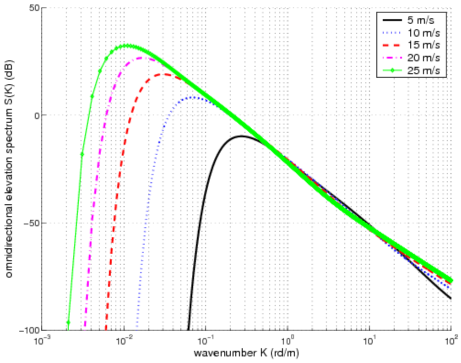

Figure 1: Elfouhaily sea surface spectra with different wind speeds (isotropic component).

To generate a realistic ocean surface associated to a given weather condition (wind speed and wind direction), we introduce the sea spectrum developed

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by Elfouhaily et al. [7] since it is very consistent with experimental data. This sea spectrum is in the form:

$$S(K,\,\phi) = M(K)f(K,\,\phi) \tag{5}$$

where $M(K)$ represents the isotropic part of the spectrum modulated by the angular function $f(K,\,\phi)$, and where K and ϕ are respectively the spatial wave number and the wind direction, see figure (1). Then, the convolution of this spectrum with an unitary white Gaussian random signal generates a one-dimensional profile (a statistical realization for the sea surface) that represents an ocean surface for given weather conditions (see fig. 2).

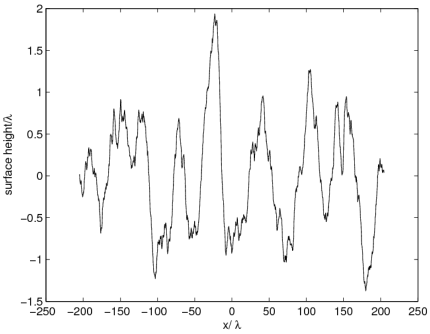

Figure 2: Example of an ocean surface profile generated where the wind speed is $10\,m/s$.

To introduce the movement of the so-generated random profiles, we must take into account the velocity of the sea waves that depends on the wave number [8]. The velocity of the longer wavelength waves is mainly influenced by the gravity, whereas for the shorter wavelength waves the predominant effect in the velocity is the capillary. For non-shallow water, the dispersion relation can be approximated by:

$$\omega\,(k) \approx k \cdot \sqrt{\frac{g}{k} + \frac{\tau k}{\rho}} \tag{6}$$

where g is the gravitational acceleration, τ is the water surface tension and ρ is the density.

3.2 Electromagnetic scattering simulations

To compute the electromagnetic field scattered by the ocean-like profiles previously generated, we apply an accelerated Method of Moments (MoM). Then, using the so-computed scattered fields, we can numerically simulate the temporal stochastic process corresponding to the RCS for different weather conditions. In this way, the numerical data generated by a Monte-Carlo methodology can be seen as the realizations of the stochastic process related to the electromagnetic scattered field.

4 Conclusion

Using these computed data, an efficient algorithm developed by Levanony and al. [9] is applied to identify the parameters of the nonlinear SDEs (4).

Finally, the parameters of the dynamic stochastic model can be associated to the different sea states (wind speed and wind direction).

References

[1] A. Ishimaru, *Wave propagation and scattering in random media.* IEEE Press, 1997.

[2] Y. Ayari, A. Khenchaf, and A. Coatanhay, “Bistatic Scattering by Sea Surface at Large Incident Angles : Two Scale Model Using the Unified Sea Spectrum,” *J. Appl. Remote Sens.*, vol. 1013532, pp. 1–19, 2007.

[3] K. Ward, R. Tough, and S. Watts, *Sea Clutter: Scattering, the K Distribution and Radar Performance.* Radar, Sonar, Navigation and Avionics, Institution of Engineering and Technology, 2006.

[4] T. R. Field and R. J. Tough, “Diffusion processes in electromagnetic scattering generating k-distributed noise,” *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, vol. 459, no. 2037, pp. 2169–2193, 2003.

[5] T. R. Field, *Electromagnetic scattering from random media.* Oxford Science Publications, 2009.

[6] B. Oksendal, *Stochastic differential equations. An Introduction with Applications.* Springer-Verlag, 6th ed., 2014.

[7] T. Elfouhaily, B. Chapron, K. Katsaros, and D. Vandemark, “A unified directional spectrum for long and short wind-driven waves,” *Journal of Geophysical Research*, vol. 102, no. C7, pp. 15781–15796, 1997.

[8] J. Miranda, A. Camps, J. Gmez, M. Vall-llossera, and R. Villarino, “Time-dependent sea surface numerical generation for remote sensing applications,” in *IGARSS’05*, 2005.

[9] D. Levanony, A. Shwartz, and O. Zeitouni, “Recursive identification in continuous-time stochastic processes,” *Stochastic Processes and their Applications*, vol. 49, no. 2, pp. 245 – 275, 1994.