

# Modeling of Electromagnetic Waves Scattering From Sea Surface Using Higher-Order Moment Method (HO-MoM) and NURBS Patch

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**Abstract** — Radar Cross Section (RCS) from sea surface carries rich informations about sea states. Many works have been developed to compute RCS from this surface. The Classical Moment Method became an effective tool since it works for a very large class of surface shapes and scattering mechanisms. However, it requires a small mesh step ( $\lambda/10$ ) which produces a high number of unknowns. Besides, for a highly curved surface like the sea, we need a much smaller mesh step to avoid the factitious geometric discontinuities between the patches. In this work, we propose to use the Higher-Order Moment Method (HO-MoM) with NURBS patch to increase the mesh step into  $\lambda$ . We show that this method can solve the scattering problem with fewer number of unknowns.

## 1 Introduction

Electromagnetic waves scattering from sea surface carry rich information about sea states: the height of sea waves, the speed and the direction of the wind, the presence of pollutants and ships etc. Due to the complexity, we can not have the exact analytical solution of the scattering waves from sea surface. The existing methods are classified into: asymptotic and numerical methods.

The asymptotic methods are still in great interest thanks to their small computation time. They can be used to predict the physical phenomena in real time. However, these methods are valid for certain type of surfaces and scattering mechanisms, but neglect the contribution of other scattering mechanisms and thus are not accurate in general [3]. The most commonly used asymptotic methods are: Kirchhoff Approximation (KA), Small Perturbation Method (SPM) and Two Scales Models (TSM).

The numerical methods in other side are very greedy in computational time. However they give more precise results and work for all scales of roughness and scattering mechanisms. Among them, the Moment Method (MoM) is a favorite tool to use in

the wave scattering area seeing that we can reduce the 3-D problem into 2-D. The first work in this method concerns to compute the surface electromagnetic current due to the incident wave. Once the current is found, we can trace the RCS and the electromagnetic fields everywhere in the space.

Most of the works in sea scattering computation by MoM use the zero order basis function (Classical MoM). However, we can meet two major obstacles that produce a big unknown number of the equations.

- electromagnetic obstacle: Classical MoM requires the mesh size step of  $\lambda/10$
- geometry obstacle: for a highly curved surface, we need smaller mesh step to avoid the factitious discontinuities between the patches

In this paper, we will use the Higher-Order Moment Method with NURBS patch to overcome these obstacles. In the first time we construct the sea surface profiles from the Elfouhaily's spectrum. In the second time, we speak about the NURBS patch and its advantage to represent the surface in a small number of patches. In the third time, we discuss the HO-MoM, and in the last time, we do some simulation using this method for a flat and a sea surface profile.

## 2 Sea Surface Modeling

Sea surface profiles are the random physical systems whose evaluations are controlled mainly by the wind and the gravity. Their analytical representation is given by the spectrums: Gaussian, Pierson-Moskowitz, Elfouhaily etc. The Elfouhaily's spectrum is adopted in our problem seeing that it is very consistent with the experimental data. This sea spectrum is given in the form [2]:

$$S(K, \phi) = M(K)f(K, \phi) \quad (1)$$

$M(K)$  represents the isotropic part of the spectrum. This spectrum is modulated by the angular function  $f(K, \phi)$ .  $K$  and  $\phi$  are respectively the spatial wave number and the wind direction.

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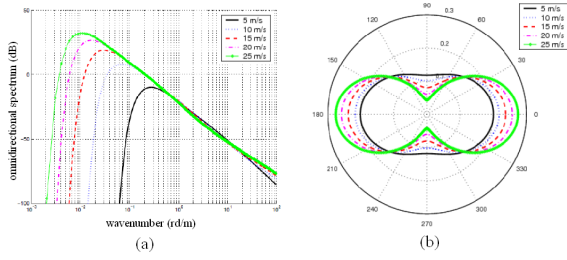


Figure 1: Elfouhaily's spectrum (a) Omnidirectional spectrum (b) Angular function

The sea surface profiles for a given wave speed and direction are obtained by the convolution of Elfouhaily's spectrum with 2D white Gaussian signal. The surface roughness increases as the wind speed. As for the wind direction, it gives the maximum impact when  $\phi = 0^\circ$  and minimum for  $\phi = 90^\circ$ . A surface profile generated in the wind speed  $U = 10 \text{ m/s}$  and the direction  $\phi = 0^\circ$  is given in figure (2).

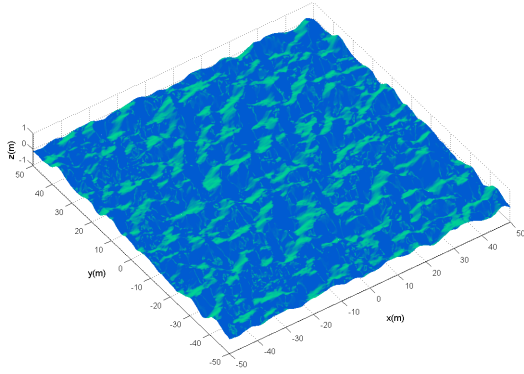


Figure 2: Sea surface profile generated from Elfouhaily spectrum for  $U = 10 \text{ m/s}$  et  $\phi = 0^\circ$

### 3 NURBS Patch

The first step in the Moment Method application concerns to represent numerically the geometry of the surface. The basic model is to mesh the surface as the sum of the planar triangle or quadrilateral patches. The patch number depends on the surface shape. For a highly curved surface, we need more patches to avoid the factitious geometric discontinuities between between the patches. The Non-Uniform Rational Basis Spline (NURBS) can be used to decrease the mesh number.

NURBS is a mathematical model commonly used in computer graphics for generating and representing curves and surfaces which offers great flexibility and precision. International Standardization Organization (ISO) selected it as the only mathematical method to define the shaps of industrial product

[5]. The use of the NURBS in the electromagnetic scattering problem was introduces firstly time by the Spanish scientists [7].

Each point of the NURBS patch is computed by taking a weighted sum of a number of control points. The control points determine the shape of the curve. Figure (3) shows the technique to construct a NURBS patch.

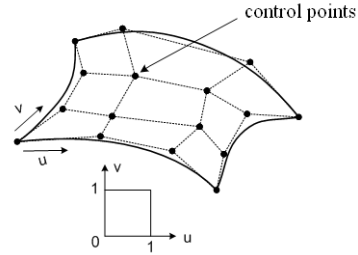


Figure 3: Nurbs construction

The mathematical expression for NURBS patch is:

$$\bar{r}(u, v) = \frac{\sum_{i=0}^m \sum_{j=0}^n w_{ij} \bar{P}_{ij} B_i^m(u) B_j^n(v)}{\sum_{i=0}^m \sum_{j=0}^n w_{ij} B_i^m(u) B_j^n(v)} \quad (2)$$

$\bar{P}_{ij}$  are the control points,  $w_{ij}$  are the weights, and  $B_i^m(u)$  are the Bernstein polynomials of degree  $m$  defined as:

$$B_i^m(u) = \frac{m!}{i!(m-i)!} u^i (1-u)^{m-i} \quad (3)$$

where  $u, v \in [0, 1]$

The NURBS can preserves the surface normal and radius of curvature at every point by adjusting the control points. Figure (4) shown the representation of the sphere with three types of the mesh. We notice that with the smallest patch number, NURBS model represents the best mesh quality compared to the triangle and quadrilateral planar patches.

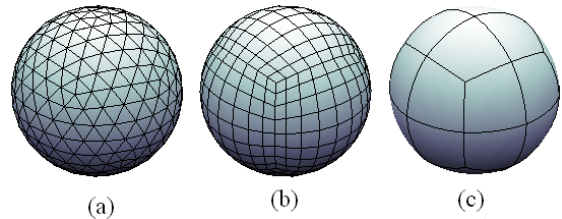


Figure 4: Mesh representation of the sphere (a) 500 triangle mesh (b) 384 quadrilateral mesh (c) 24 NURBS

## 4 High Order Moment Method

The sea surface can not be summed up to a geometric discription. From the electromagnetic point of view, salt sea water can be considered as a perfect conductor since its permittivity is large enough. We can model it in the Electric Field Integral Equation (EFIE) or the Magnetic Field Integral Equation (MFIE). In this work, we choose the EFIE, expressed in:

$$\hat{n} \times \bar{E}^{inc} = \hat{n} \times \left[ j\omega\mu \int_S \bar{J}_S(\bar{r}') G(\bar{r}', \bar{r}) dS' - \frac{1}{j\omega\epsilon} \nabla \int_S \nabla'_S \cdot \bar{J}_S(\bar{r}') G(\bar{r}', \bar{r}) dS' \right] \quad (4)$$

$\bar{E}^{inc}$  is the incident wave,  $\bar{J}_S(\bar{r})$  is the surface equivalent current and  $G(\bar{r}', \bar{r})$  is the Green function::

$$G(\bar{r}', \bar{r}) = \frac{e^{-jk|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|} \quad (5)$$

where  $k = \omega\sqrt{\epsilon\mu} = 2\pi/\lambda$  is the vacuum wavenumber and  $\bar{r}'$  and  $\bar{r}$  are the source and observation points respectively.

The surface current  $\bar{J}_S(\bar{r})$  is described in the arbitrary coordinate  $u$  and  $v$ .

$$\bar{J}_S(\bar{r}) = J_S^u(\bar{r})\bar{a}_u + J_S^v(\bar{r})\bar{a}_v \quad (6)$$

where  $\bar{a}_u$  and  $\bar{a}_v$  are the co-variant unitary vectors:

In the high order moment method, these currents are expanded in the form [4]:

$$\bar{J}_S^u(\bar{r}) = \frac{1}{Ja(u,v)} \sum_{m=0}^M \sum_{n=0}^N b_{mn}^u f_{mn}^u(u,v) \quad (7)$$

$$\bar{J}_S^v(\bar{r}) = \frac{1}{Ja(u,v)} \sum_{m=0}^M \sum_{n=0}^N b_{mn}^v f_{mn}^v(u,v) \quad (8)$$

where  $Ja(u,v) = |a_u \times a_v|$  is the surface Jacobian,  $b_{mn}^u$  and  $b_{mn}^v$  are unknown coefficients, and  $f_{mn}^u(u,v)$  and  $f_{mn}^v(u,v)$  are the Legendre polynomials. Since in the Legendre polynomials  $u, v \in [-1, 1]$ , we have to modify the NURBS equation (2) to be used with the HO-MoM [5].

## 5 Simulation

In a very small wind speed condition, the sea surface is seen as a flat. We will start the simulation for this surface to show the advantage of the HO-MoM. The RCS calculated from Classical MoM are compared with the RCS from HO-MoM. The base function of the Classical MoM is the pulse function [6].

### 5.1 RCS of plat surface

A plate surface with the length  $4\lambda \times 4\lambda$  is taped by plane incident wave (wave-length =  $\lambda$ ). In the classical moment method, we mesh this surface in  $\lambda/10$  mesh step. If  $Q$  = mesh number, the unknown number of this systems is  $3 \times Q$  since we introduce a three coordinate system ( $xyz$ ) in each mesh. For the HO-MoM, we construct the NURBS patch with the length  $\lambda$  and the unknown number  $2 \times (M + 1) \times (N + 1) \times Q$ .

Figure (5) shows the RCS from flat surface computed by Classical MoM and HO-MoM. HO-12 define the order  $M = 1$  and  $N = 2$  respectively. We notice that with ten times larger mesh step, HO-MoM order-22 and order-33 can approach the Classical MoM.

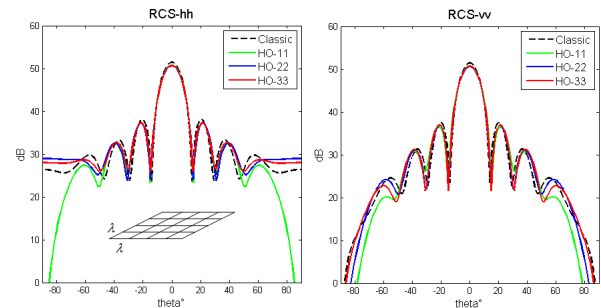


Figure 5: RCS from a  $4\lambda \times 4\lambda$  flat

To study the accuracy of the HO-MoM in the function of the base function order, we use the root-mean-square (RMS) error of the RCS [9], defined as:

$$RMS = \sqrt{\frac{1}{N_s} \sum_{i=1}^{N_s} |\sigma_{HO} - \sigma_{classic}|^2} \quad (9)$$

since we have two polarization type ( $hh, vv$ ), we compute the average of them by:

$$(RMS_{hh} + RMS_{vv})/2 \quad (10)$$

Table (1) summarizes the comparison between Classical MoM with the HO-MoM.

Method	patch length	unknown	error(dB)
Classic	$\lambda/10$	4800	-
HO-11	$\lambda$	128	10.9
HO-12	$\lambda$	192	6.3
HO-22	$\lambda$	288	2.9
HO-23	$\lambda$	384	2.1
HO-33	$\lambda$	512	1.9

Table 1: Classical MoM and HO-MoM comparison

## 5.2 RCS of Sea surface

Since the sea surface is a random physical system, we can generate many profiles with the same wind speed and direction. As an example, we generate a  $10\text{ m} \times 10\text{ m}$  for  $U = 10\text{ m/s}$  and  $\phi = 0^\circ$  as shown by figure (6). This surface is taped by a plane wave ( $\lambda = 1\text{ m}$ ). To use the Classical MoM, we divide this surface into 10,000 meshes (mesh step =  $0.1\text{ m}$ ). For the HO-MoM with NURBS patch, the surface is divided into 100 patches (patch step =  $1\text{ m}$ ).

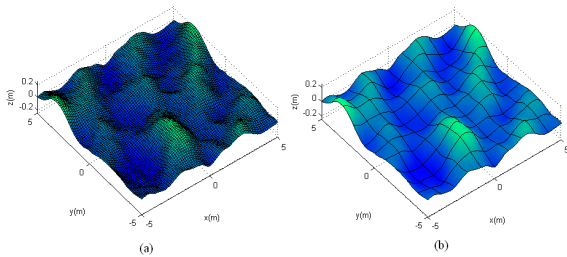


Figure 6: A sea surface profile for  $U = 10\text{ m/s}$  and  $\phi = 0^\circ$  (a) 10,000 quadrilateral mesh (b) 100 NURBS patch

Radar Cross Section from this profile computed by Classical MoM and HO-MoM are shown in figure (7)

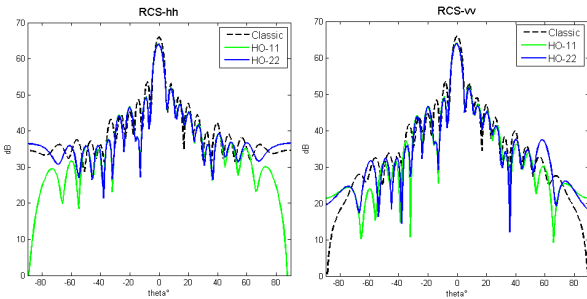


Figure 7: RCS from sea profile in fig (6)

## 6 Conclusion

Ho-MoM have ever been applied in this context [8], but this paper shown the advantage of HO-MoM with NURBS model to solve the electromagnetic wave scattering from a flat and a sea surface profile. A pure plane wave is used in this work. In numerical simulation, the surface must be limited at the area of  $Lx \times Ly$ . This means that the surface current is forced to be zero in the edge and produce the artificial reflection. To solve this problem we can use the tapered incident wave developed by Braunsch [1]. Besides, the simulations have been done for only one sea surface profile. In future studies,

we will simulate the scattering wave from many surface profiles (Monté-Carlo technique) to statistically analyze the RCS.

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