Kalman Filtering of GPS Signals for Target Tracking Application

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***Abstract*—Considering the GPS link as a passive sensor for ocean monitoring, we study the possibility of tracking the GPS signal reflexion footprints on sea surface to improve the acquisition of this signal. As an analogy with the classical problem of moving targets Radar tracking, we develop a tracking algorithm based on Kalman filtering. Our study is based on calculated and measured values of Doppler frequency shift and time delay.**

I. Introduction

The Global Navigation Satellite System (GNSS) presents a powerful and a useful technology for teledetection, ocean surface monitoring and oceanography. Many experiences have been held to show the efficiency of the Global Positioning System (GPS) in applications such as ocean surface altimetry, wave height, surface current measurements, and current direction estimation [1]. That is the reason why, in the project MOPS [2] (Marine Opportunity Passive Systems), we are interested in studying the feasibility of passive systems, such as GPS and Galileo, in the vicinity of sea surface for oceanographers. Actually, we aim to develop a platform dedicated to collect raw data then to process these data to extract useful information. In [3], we have already presented our own approach in using the GPS signals for near-surface applications and costal observations or monitoring. Concretely, we defined an experiment that deals with interaction between an electromagnetic wave and some mobile target. The target being detected, we will extend the problem of interaction to track the trajectory of some mobile target in a defined environment. Since we are dealing with long term GPS signals, weak targets and non-stationary medium, an adaptive filtering is required.

II. Problem formulation

In this work, we study the possibility of tracking the GPS signal reflexion footprints on sea surface for a better acquisition of this signal. These reflexion points have noise corrupted positions and depend mainly on sea state. The modelisation and the initialisation of parameters are possible with simple and realistic hypothesis on sea surface. We consider that the signal is reflected by the sea wave near the cost. In this case, the moving target is supposed to be the crest of a wave approaching the cost. The problem dimensions are perfectly determined by the position of the emitter which is a GPS satellite, the receiver which is an antenna, operating in a passive mode, located at the top of a light house at 22m as it is shown in the figure 1.

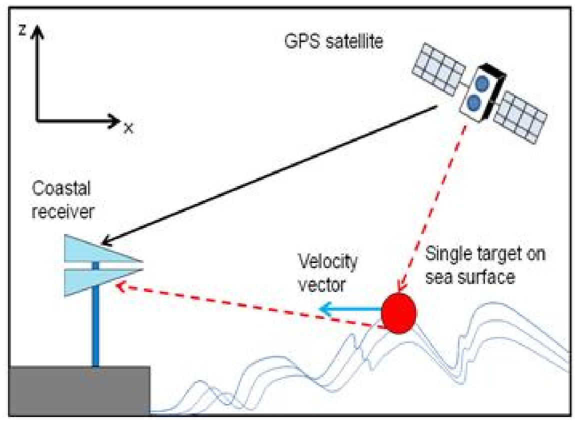

Figure 1. Scheme of the environment

This antenna is a part of a measure platform that allows processing the scattered electromagnetic waves. And finally by the target which is defined by its motion equations. In order to achieve this work, we suppose an analogy with the classical problem of moving targets tracking and develop a tracking algorithm based on Kalman filtering.

III. Tracking filter

The role of tracking filter is to carry out recursive target state estimation given: the target dynamic equation, the sensor measurement equation, and the target originated measurements. The tracking filter considered here is the Kalman filter which is presented in [4]. As an illustration, let be the mobile target (wave crest). It has a constant velocity depending on the gravity acceleration g, the water depth h and the length λ_{water} of the sea wave itself. The velocity v can be calculated as shown in this equation:

$$v = \sqrt{\frac{g\lambda_{water}}{2\pi}\tanh\left(\frac{2\pi h}{\lambda_{water}}\right)} = 1.7661 \qquad (1)$$

where $g = 9.80m/s^2$, $\lambda_{water} = 2m$, and $h = 10m$. The target is moving according to its motion equations: $x = 1500 + vt$ and $z = -22\,m$.

In our tracking approach, we consider estimating the values of the Doppler frequency shift δ due to the target motion and time delay τ. To perform this filtering, let us define our system parameters:

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A. System dynamics

The state vector can be defined as: $s = \left[\begin{array}{cc} \delta & \tau \end{array} \right]^T$. The dynamic evolution of the system is defined by the following recursive equation:

$$s_{k+1} = f(s_k) + v_k \tag{2}$$

: Where f is a known linear function of $s_k$represented by the matrix $F = \left[\begin{array}{cc} 1 & T \\ 0 & 1 \end{array} \right]$, $v_k \sim N(0, \sigma_v^2)$. $T = 1ms$ is the revisit time of the GPS signal.

$$\sigma = -\frac{f_c}{c} \cdot \frac{x_k \dot{x}_k + z_k \dot{z}_k}{\sqrt{x_k^2 + z_k^2}} \tag{3}$$

$$\tau = \frac{\sqrt{x_k^2 + z_k^2}}{c} \tag{4}$$

where $f_c = 1.5742\, GHz$, the carrier frequency of the GPS signal, and $c = 3.10^8 m/s$.

B. Measurement model:

The measurements are related to the target state by the following equation:

$$z_k = g(s_k) + w_k \tag{5}$$

Where g is a known linear function of s_k and $w_k \sim N(0, \sigma_w^2)$. Actually, the values of s_k are extracted from an independent experiment presented in [3] dedicated to measure the Doppler frequency shift and the time delay of the GPS reflected signal.

In this experiment, we consider not only the target itself but also its interaction with the satellite and the receiver. Here, every range is considered.

Let the letters R, S, C and F represent respectively the receiver located at (0,0,0), the GPS satellite (15000,15000, 20200000), the moving target (x,y,z) and the fixed target (5000, 0, -22).

In this case, the Doppler is calculated with respect to both ranges RC and SC, and the time delay is calculated with respect to the ranges SC, CR and SR. Thus $\delta = -\frac{f_c}{c} \cdot \frac{d}{dt}(RC + CS)$ and $\tau = \frac{SC + CR - SR}{c}$.

These values are used to determine the initial position in Doppler/Delay plan at the instant t =0 and the evolution in time and frequency of the signal reflected of the moving target parameters.

The observation or measurement values are extracted from the Doppler/Delay maps noise corrupted obtained after correlation of the received signal with a pure replica [3]. Actually, we use an acquisition algorithm with shifting replica in DFT domain described in [5]. Once these values are estimated, then they are injected as an observation vector necessary to perform the filtering according to the Kalman equations [4].

IV. Filtering results

The observed values allow us to track the position of the moving target at every instant t for a duration of 100 ms. Actually, we believe that it is more interesting to have a finest estimation of the Doppler and time delay values rather than trajectory estimation itself.

The following figures show the computation of the Doppler and the time delay through the different steps to have a smoothed Doppler/Delay Map (DDM). The Doppler value shown on the y-axis is $\triangle_{Doppler}$ which is equal to the difference between the estimated value of the Doppler δ (after filtering) and the mean of all the Doppler obtained during 100 ms.

$$\delta = \triangle_{Doppler} - 9.2668056455 \tag{6}$$

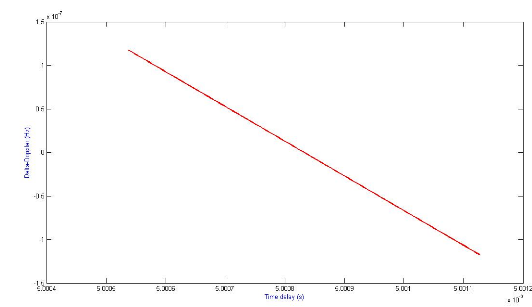

Figure 2. Real values of Doppler Map DDM

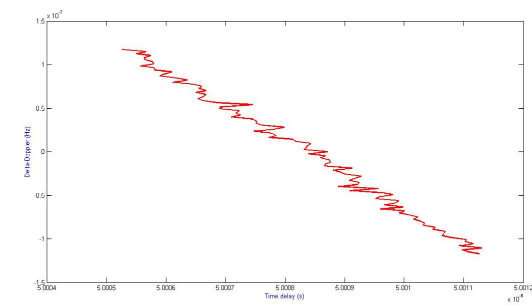

Figure 3. DDM resulting from noise corrupted observation

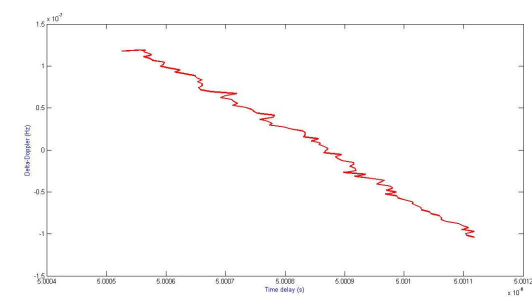

Figure 4. DDM after Kalman filtering

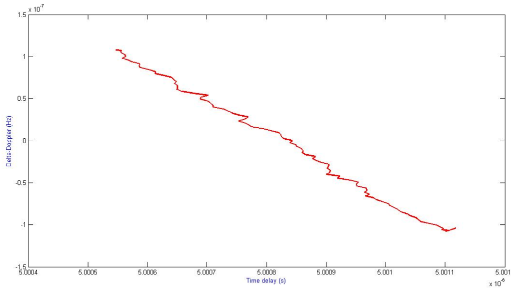

Figure 5. Smoothed DDM

V. Conclusion and perspectif

The main objective of this paper is to join the two approaches: tracking targets and long coherent summations, in order to extract the GPS signals reflection on the sea surface. In order to accomplish this work, we used a Kalman Filtering for single weak target, allowing us to obtain the best estimation possible especially when the functions f and g used are linear because the algorithm in this case is converging. In the future, we will use a Track-before-Detect algorithm in order to track the power spectral density from one summation to another for multiple target scenarios. We propose to use a particle filter for the TBD. Actually, the particle filter is one of the best used algorithms for TBD especially for root mean square (rms) error position estimation and multitargets detection [6].

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