The Observability of Marine Weather Conditions Using the Electromagnetic Waves Scattering from the Sea Surface: A Statistical Analysis

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Abstract — **The aim of this work is to quantify the observability of marine weather conditions using the bistatic Normalized Radar Cross Section (NRCS) from the sea surface. The mathematical model of the sea surface profiles is obtained from the Elfouhaily's spectrum and the NRCS are calculated using the Method of Moments (MoM). All of NRCS are recorded in some data sets depending on the weather conditions. Using the the T-value and the Linear Discriminant Analysis, we evaluate the contrast of the data sets, meaning the observability of the weather conditions as the function of the incident angle, scattering angle and the polarization types (hh, vv, vh, hv).**

1 INTRODUCTION

The electromagnetic waves scattering from the sea surface carries rich information in the ocean monitoring activity. By analyzing its Normalized Radar Cross Section (NRCS), one can draw the wave height, the marine weather conditions (the speed and the direction of the wind above the sea surface), and also the presence of the unknown objects on the surface.

In this study, we want to quantify the observability (the ability to distinguish some data sets) of the marine weather conditions as the function of the incident angle, scattering angle and the polarization types (hh, vv, vh, hv). This information is important especially to the installation of the measuring instrument (for instance [1]). To achieve this goal, this work is divided into three parts.

In the first part, several realistic sea surface profiles for some weather conditions are generated by using the convolution of the Elfouhaily's spectrum with 2D white Gaussian random signal.

In the second part, we calculate the NRCS from these surface profiles using the Method of Moments (MoM). The NRCS are classified into the sets of data depending on the weather conditions and then we find the mean value and the standard deviation for each data set with the Monte Carlo approach.

Finally, using the contrast criterion, we evaluate the contrast of the sets of data, meaning the observability of the weather conditions. When the observation is based upon only one NRCS coefficient, we use the mono-dimensional contrast criterion like T-value. But when we record the four polarizations simultaneously, we introduce a multivariate statistical analysis, the Linear Discriminant Analysis (LDA).

2 SEA SURFACE MODELISATION

The Sea surface profiles can be considered as the random physical systems whose evaluations are controlled mainly by the wind and the gravity [3]. Their analytical representation is often given by the spectrums: Gaussian, Pierson-Moskowitz, Elfouhaily etc. The Elfouhaily's spectrum [2] is adopted in our problem seeing that it is very consistent with the experiment data [7]. This sea spectrum is given in the form:

$$S(K, \phi) = M(K) f(K, \phi) \tag{1}$$

where $M(K)$ represents the isotropic part of the spectrum modulated by the angular function $f(K, \phi)$, and K and ϕ are respectively the spatial wave number and the wind direction.

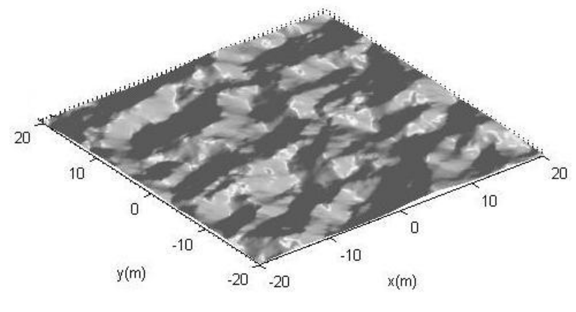

Figure 1: Sea surface profile for $U = 10\,\mathrm{m/s}$ and $\phi = 0°$

The sea surface profiles for a given weather condition are obtained by the convolution of Elfouhaily's spectrum with 2D white Gaussian random signal. The surface roughness increases as the wind speed. As for the wind direction, it gives the maximum

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impact when 0° and minimum for 90°. A surface profile generated in the weather condition of the wind speed $U = 10\,\mathrm{m/s}$ and the direction 0° is given in figure (1).

3 CALCULATION OF NRCS

There are various methods to calculate the NRCS from the sea surface. The approximation methods (ex: Small Perturbation Method, Kirchhoff Approximation etc) can provide only a reliable estimation of the mean value of NRCS. However, to quantify the observability, we need more statistical information like standar deviation. Therefore we propose to use a numerical method (Method of moments -MoM) [4, 5].

The sea surface can be considered as a perfect conductor since its permittivity is large enough. We can modelize it in the Electric Field Integral Equation (EFIE) or the Magnetic Field Integral Equation (MFIE). We chose the MFIE because it usually results in a better numerical condition [5]. The general form of the MFIE is given by:

$$\frac{\hat{n} \times \overline{H}(\overline{r})}{2} = \hat{n} \times \overline{H}_{inc}(\overline{r}) + \hat{n} \times \int dS' \nabla g(\overline{r}, \overline{r}') \times \hat{n}' \times \overline{H}(\overline{r}') \tag{2}$$

where $g(\overline{r}, \overline{r}')$ is the Green's function and $\overline{H}_{inc}(\overline{r})$ is the incident wave. In numerical simulation, the surface is limited at the area $L_x \times L_y$. This means that the surface current is forced to be zero in the edge and it will produce the artificial reflection. To avoid this problem, one way is to taper the incident wave so that it decays to zero in a Gaussian manner. For an incident wave centered in direction $\hat{k}_i = k\left(\sin\theta_i \cos\phi_i \hat{x} + \sin\theta_i \sin\phi_i \hat{y} + \cos\theta_i \hat{z}\right)$ with wave number k, the tapered incident wave $\overline{H}_{inc}(\overline{r})$ is given by:

$$\overline{H}_{inc}(\overline{r}) = \frac{1}{\eta} \int_{-\infty}^{\infty} d\overline{k}_\rho \, e^{i(\overline{k}_\rho \cdot \overline{\rho} - k_z z)} \psi(\overline{k}_\rho) \overline{h}(\overline{k}_\rho) \tag{3}$$

where

$$\overline{k}_\rho = \hat{x} k_x + \hat{y} k_y, \qquad k_z = \sqrt{k^2 - k_x^2 - k_y^2} \tag{4}$$

η is the characteristic impedance of the wave, $\psi(\overline{k}_\rho)$ is the spectrum of the incident wave that carries the information on the shape of the footprint and also the direction of the wave, and $\overline{h}(\overline{k}_\rho)$ is the polarisation vector [6].

The integral equation (2) is then solved with a point matching Method of Moments. Using pulse basis function and delta testing function, this equation is cast into the matrix equation:

$$\overline{\overline{Z}} \overline{x} = \overline{b} \tag{5}$$

A standard linear equation resolution method can be used to solve this equation so that we can calculate the scattering field from the ocean surface.

For the far-field observation, one is interested especially in the term of Normalized Radar Cross Section (NRCS):

$$\sigma_{\alpha\beta} = \lim_{R \to \infty} \frac{4\pi R^2 \left| H^s_{\alpha\beta} \right|}{A \left| H^{(i)}_\beta \right|^2} \tag{6}$$

where $\alpha, \beta = h, v$ represent the polarization of scattered and incident waves respectively, R is the distance of the target to the observation points, A is the geometric area of the target, and H^s and $H^{(i)}$ are the scattered and incident magnetic field amplitude. In the 1D surface probleme, there are only the terms co-polarization of NRCS $(\sigma_{hh}, \sigma_{vv})$, but for our study (2D surface), we will have four NRCS terms $(\sigma_{hh}, \sigma_{vv}, \sigma_{vh}, \sigma_{hv})$ according to the co-polarization and cross-polarization.

4 DATA CONTRAST CRITERION

The observability of data is defined as the ability to distinguish some data sets. In figure (2), we show two data set distributions. The data set 1 and data set 2 in the second distribution are better separated than in the first one. In the statistical term, the observability of the second distribution is stronger than the first one.

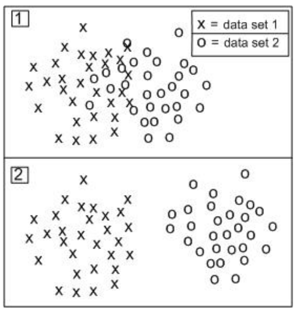

Figure 2: The data set distributions for two conditions: (top) weak observability (buttom) strong observability

There are many possible techniques to quantify the observability of the data. If the data sets are mono-variable, we can use the T-value contrast criterion. When they are multi-variable, we must introduce a multi-variate statistical analysis like the Linear Discriminant Analysis (LDA).

4.1 T-value contrast criterion

Be two sets of mono-variate data, data set 1 and data set 2. The T-value contrast criterion for these data sets is given by [8]:

$$T(\theta) = \frac{|m_1(\theta) - m_2(\theta)|}{\sigma(\theta)} \tag{7}$$

$$\sigma(\theta) = \sqrt{\left[\sigma_1^2(\theta) + \sigma_2^2(\theta)\right]/n_r} \tag{8}$$

where m_1 and m_2 are the mean values, σ_1 and σ_2 are the standard deviations for two data sets. n_r is the number of data for each data set. The T-value is directly related to the Student-test and can be easily translated in term of good or false detection rate.

4.2 Linear Discriminant Analysis

In a statistical point of view, the two data sets of n-dimension represent the two labeled classes respectively denoted by Y_1 and Y_2. The Linear Discriminant Analysis consists in finding the projection space that best separates two classes $p\left(X|Y = Y_1\right)$ and $p\left(X|Y = Y_2\right)$. LDA commonly relies on Fisher's Linear Discriminant and assumes the probability density function are Gaussian with means $\overline{Y}_1$ and $\overline{Y}_2$ with the same covariance Γ. Fisher's discriminant S is the ratio of the two variances between and within the two classes Y_1 and Y_2 after been projected along the vector $\overrightarrow{\varpi}$:

$$S = \frac{\sigma^2_{between}}{\sigma^2_{within}} = \frac{\left(\overrightarrow{\varpi} \cdot \overline{Y}_1 - \overrightarrow{\varpi} \cdot \overline{Y}_2\right)^2}{2\overrightarrow{\varpi}^T \cdot \Gamma \cdot \overrightarrow{\varpi}} \tag{9}$$

It can be shawn that the vector $\overrightarrow{\varpi}$ that maximized the Fischer ration

$$\overrightarrow{\varpi} = \frac{1}{2}\Gamma^{-1} \cdot \left(\overline{Y}_1 - \overline{Y}_2\right) \tag{10}$$

In practical point of view, the LDA provides two elements. In one point, the Fisher's discriminant is an evaluation of the statistical distance between two data sets. In this sense, the Fisher's discriminant can be seen as direct mesurement of the observability. Another point is that the LDA also provides a vector that represents the optimal projection in the n-dimensional space to separate two classes.

5 NUMERICAL SIMULATION

5.1 Sea surface and incident wave

The application of the numerical method is imposed by limiting the sea surface dimension. In this study, we take the dimension $40\,\text{m} \times 40\,\text{m}$, discretized by the surface of $1\,\text{m} \times 1\,\text{m}$. It is logically enough to see the influence of the wind speed $5\,\text{m/s}$, $7\,\text{m/s}$ and $10\,\text{m/s}$. We realize 1000 surfaces for each weather condition.

As for the incident wave, we take the one with the wave length $4\,\text{m}$ (frequency of $1,6\,\text{MHz}$) at the normal incidence in the plane of propagation (θ_s between $-90°$ and $+90°$).

5.2 Bistatic Normalized Radar Cross Section (NRCS)

The four bistatic NRCS $(\sigma_{hh}, \sigma_{vv}, \sigma_{vh}, \sigma_{hv})$ are calculated for each surface realisation, we get then the NRCS of 1000×4 dimension for each weather condition. Using the Monte Carlo technique, we calculate the average of these NRCS. The influence of the wind speed to the NRCS for all polarization in normal incident angle is shown in figure (3).

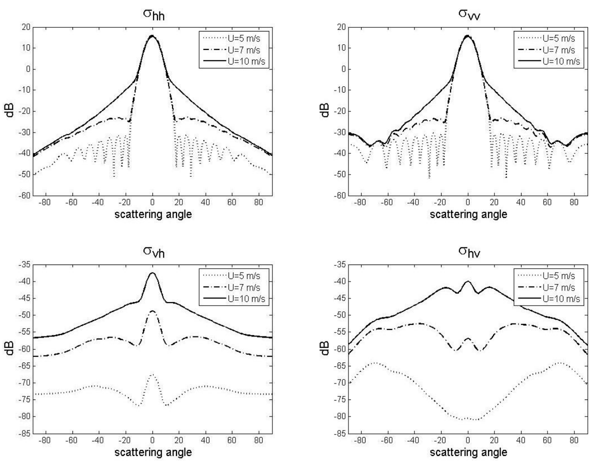

Figure 3: The mean of the Bistatic NRCS in the function of the scattering angle obtained by Monte Carlo technique with normal incident wave (0°)

For all polarization types, we note that NRCS are maximal in the specular scattering direction. The co-polarization NRCS $(\sigma_{hh}, \sigma_{vv})$ values decrease about specular direction and increase for other angle when the wind speed increase. The cross-polarization NRCS $(\sigma_{vh}, \sigma_{hv})$ increase for all scattering angle, but their values are very small compared to the co-polarization NRCS.

5.3 Monovariate observability

The T-value is used to distinguish the wind speeds $5\,\text{m/s}$ versus $7\,\text{m/s}$ (T-1) and $7\,\text{m/s}$ versus $10\,\text{m/s}$ (T-2) with the same wind direction ($\phi = 0$) for four polarization types analized separately. Figure (4) shows the T-value for the normal incident wave.

We note that the T-values are maximal in specular direction for all polarization types. However, in

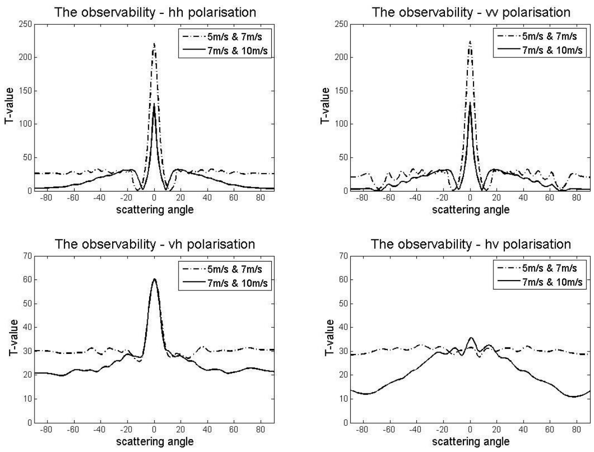

Figure 4: The observability of the weather conditions obtained by T-value contrast criterion for four polarization types as the function of the scattering angle with normal incident wave (0°)

co-polarization types, the minimum points are located close to the specular direction and will bring a serious problem to the installation of the measuring instrument. This problem does not appear in cross-polarization types. Another notes, the T-1 is relatively bigger and constant than T-2. T-2 decrease a near by low rasant angle.

5.4 Multivariate observability

The assumption that the receiver perceives only one parameter (only one polarization) has a strong limitation to evaluate the bistatic systems. The most sophisticated observation systems can record the four polarizations simultaniously. In these conditions, we use the Fisher's Discriminant Criterion obtained from the Linear Discriminant Analysis (LDA).

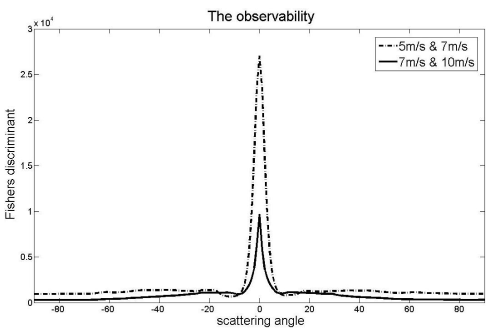

Figure 5: The observability of the weather conditions obtained by LDA as the function of the scattering angle with normal incident wave

Figure (5) shows the observabily obtained by LDA for wind speed $5\,\mathrm{m/s}$ versus $7\,\mathrm{m/s}$ (LDA-1) and $7\,\mathrm{m/s}$ versus $10\,\mathrm{m/s}$ (LDA-2). We note that their value is maximal in the specular direction and there is no minimum point close to it. LDA-1 is bigger and constant than LDA-2 for all scattering angle.

6 CONCLUSION

We have done a statistical analysis to obtain the observability of the weather conditions. For all simulations, we note that the observabiliy is maximal in the specular scattering direction. However, there are the minimum points close to specular direction in the co-polarization types. This problem disappears in multi-variate observability. Another note, the observability is more important and constant in the small wind speed.

All of the simulations in this work are done in the small frequency incident wave. To obtain more accurate results, we have to increase its frequency. Our target is to work in the GPS/Galileo signal frequency (L-Band).

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