Numerical modeling of the electromagnetic scattering by a sea surface: the MoM and Discontinuous Galerkin Method approaches

Arnaud Coatanhay [#] [*]

[#] *Lab. E3I2 EA-3872, ENSIETA*
2 rue Fran çois Verny, 29806 Brest cedex 9,France
[*] Université européenne de Bretagne, France
[1] arnaud.coatanhay@ensieta.fr

Abstract — **The main purpose of this paper is to compare the numerical simulation of the electromagnetic scattering by a sea surface using a MoM approach and Nodal Discontinuous Galerkin Methods (NDGM).**

1 Introduction

At first sight, the sea surface can be reduced to the interface between two open areas: the air and a highly conductive dielectric (salted water). In this context, the Method of Moment (MoM) appears as a very appropriate methodology to to numerically estimate this electromagnetic scattering. This is the reason why abundant literature have been published upon this approach in marine environment [1].

However, for strong wind conditions (breaking wave crests) or for non-linear sea waves (near the coast for instant), MoM raises significant accuracy problems [2, 3, 4, 5]. In addition, MoM can not take into account complex interactions between the air and the sea water (foam for example). In this case, the sea wave must be considered as a three-dimensional scatterer. Therefore, numerical methods based on volume decompositions (Finite Element,...) have to be investigated. In this paper, the MoM is compared , in L-Band (1.5 GHz), with a Nodal Discontinuous Galerkin Method (NDGM) that is a very adapted approach for electromagnetic simulation involving a complex geometry. In this study, this comparison is only made in a very canonical (TE) two-dimensional configuration.

2 Sea Model

Assuming the heterogeneity of the sea as negligeable, the sea model only consists in modeling the geometry of the surface and estimating the dielectric property of the salted water.

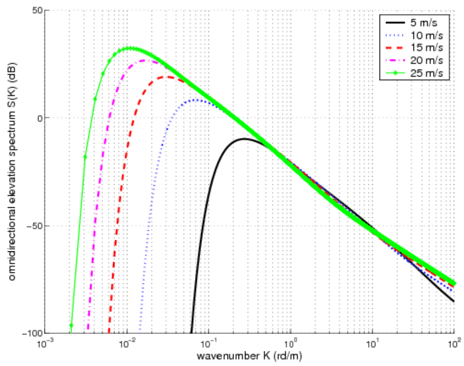


Figure 1: Elfouhaily sea surface spectra with different wind speeds (isotropic component).

2.1 Sea surface generation

To generate a realistic ocean surface associated to a given weather condition (wind speed and wind direction), we introduce the sea spectrum developed by Elfouhaily et al. [6] since it is very consistent with experimental data. This sea spectrum is in the form:

$$S(K,\,\phi) = M(K)f(K,\,\phi) \qquad (1)$$

where $M(K)$ represents the isotropic part of the spectrum modulated by the angular function $f(K,\,\phi)$, and where K and ϕ are respectively the spatial wave number and the wind direction, see figure (1). Then, the convolution of this spectrum with an unitary white Gaussian random signal generates a one-dimensional profile (a statistical realization for the sea surface) that represents an ocean surface for given weather conditions (see fig. 2).

2.2 Permittivity of the sea water

According to Debye [7], the dielectric constants of many liquids depend markedly on the frequency of mea-

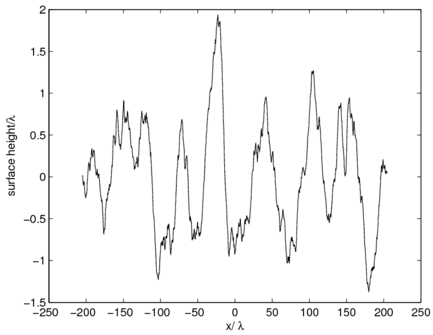


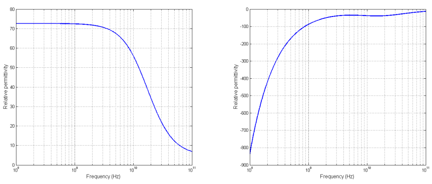
Figure 2: Example of an ocean surface profile generated where the wind speed is $10\,m/s$.

surement.

The dependence is in general found to be a decrease from static value ϵ_0 at low frequencies to a smaller limiting value ϵ_∞ at higher frequencies. In the transition region of anomalous dispersion there is an "absorption conductivity" and the situation may be described in terms of complex dielectric constant $\epsilon = \epsilon' - i\epsilon''$. Here, the sea water is assumed to be a single-pole medium with the electric susceptibility in frequency domain expressed as:

$$\epsilon = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 + j\omega\tau} \tag{2}$$

were τ is the relaxation time and ω is the radial frequency ($\omega = 2\pi f$).



(a) *Real part* (b) *Imaginary part*

Figure 3: Relative permittivity as a function of the frequency (T=20°C and S=34ppm).

In L-Band at $1.5\,GHz$, the relative permittivity is $\epsilon \simeq 72 - i\,60$, and the sea water must be considered as a very conductive medium ($\sigma_{sea} \simeq 5\,S/m$).

3 Electromagnetic scattering model

3.1 Method of Moment

As previously said, the most common approach to compute the electromagnetic field scattered by a randomly generated ocean surface that corresponds to a realization (see figure (2)), is to apply the Method of Moment (MoM) (one dimensional). A detailled presentation can be found in [8, 9]. In fact, the recent accelerated MoM such as the Forward-Backward method (FB-MOM [10, 11]) can be applied to obtain a more efficient algorithm. And a Monte Carlo process leads to the estimation of the statistical properties of the scattered field. The figure (4) presents an example of the horizontally polarized Normalized Radar Cross Section (NRCS) obtained by this way.

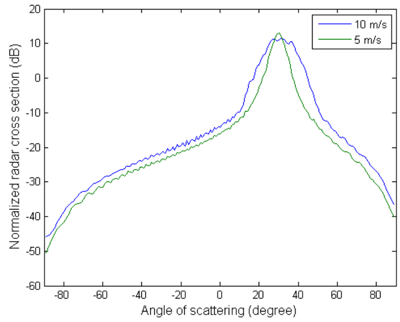


Figure 4: Numerical estimation (Method of Moment) of the radar cross section where the angle of incidence is 30 degree at 1.5 GHz.

3.2 Nodal Discontinuous Galerkin Method

In the present issue, the frequency is set to $1.5\,GHz$ and the incident field is TE ($\mathbf{E} = \mathbf{E}^z \cdot \hat{\mathbf{k}}$). So, the Mawxell equations have to be considered in this harmonic form:

$$i\omega\mathbf{H}^x = -\frac{\partial\mathbf{E}^z}{\partial y} \tag{3}$$

$$i\omega\mathbf{H}^y = \frac{\partial\mathbf{E}^z}{\partial x} \tag{4}$$

$$i\omega\mathbf{E}^z = -\frac{\partial\mathbf{H}^x}{\partial y} + \frac{\partial\mathbf{H}^y}{\partial x} \tag{5}$$

The computational domain Ω is split into elementary nonoverlapping domains D^k. In each elementary domain D^k, the Nodal Discontinuous Galerkin Method (NDGM) approximates the different electromagnetic components as the sum of local nodal polynomial functions:

$$\mathbf{x} \in D^k : \quad u\left(\mathbf{x}, t\right) = \sum_{i=1}^{N_p} u_h^k\left(\mathbf{x}_i\right) l_i^k\left(\mathbf{x}\right) \tag{6}$$

where $l_i^k\left(\mathbf{x}\right)$ is the two-dimensional Lagrange polynomial based on the grid points $\mathbf{x}_i$. $N = N_p - 1$ is the order of the model. A complete description of the theory and the applications of the NDGM are given in [12, 13, 14].

3.2.1 Boundary conditions

In the contrary of the MoM, the NDGM requires a limited computational domain, and so a Absorbing Boundary Condition (ABC) layer (Ω_{abs}) must be introduced all around the domain of interest Ω_{int} (fig. 5).

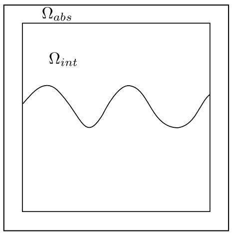


Figure 5: Computational domain including an absorbing boundary condition (ABC).

Following the method described by Abarbanel et al. [15], an artificial absorbing medium characterizes the ABC area and the maxwell equations are in the new harmonic form:

$$i\omega\mathbf{H}^x = -\frac{\partial\mathbf{E}^z}{\partial y} - \sigma^y\left(2\mathbf{H}^x + \mathbf{P}^y\right) \tag{7}$$

$$i\omega\mathbf{H}^y = \frac{\partial\mathbf{E}^z}{\partial x} - \sigma^x\left(2\mathbf{H}^y + \mathbf{P}^x\right) \tag{8}$$

$$i\omega\mathbf{E}^z = -\frac{\partial\mathbf{H}^x}{\partial y} + \frac{\partial\mathbf{H}^y}{\partial x} - \frac{d\sigma^x}{dx}\mathbf{Q}^x + \frac{d\sigma^y}{dy}\mathbf{Q}^y \tag{9}$$

$$i\omega\mathbf{P}^x = \sigma^x\mathbf{H}^y \tag{10}$$

$$i\omega\mathbf{P}^y = \sigma^y\mathbf{H}^x \tag{11}$$

$$i\omega\mathbf{Q}^x = -\sigma^x\mathbf{Q}^x - \mathbf{H}^y \tag{12}$$

$$i\omega\mathbf{Q}^y = -\sigma^y\mathbf{Q}^y - \mathbf{H}^x \tag{13}$$

where $\sigma^x = \sigma(x)$ and $\sigma^y = \sigma(y)$ define the material property of the absorbing area Ω_{abs} surrounding the area of interest Ω_{int} (see fig. 5). We let:

$$\sigma^x := \begin{cases} 0 & \mathbf{x} \in \Omega_{int} \\ \sigma_0^x \cdot \left(|x - x_{int}|^2\right) & \mathbf{x} \in \Omega_{abs} \end{cases} \tag{14}$$

$$\sigma^y := \begin{cases} 0 & \mathbf{x} \in \Omega_{int} \\ \sigma_0^y \cdot \left(|y - y_{int}|^2\right) & \mathbf{x} \in \Omega_{abs} \end{cases} \tag{15}$$

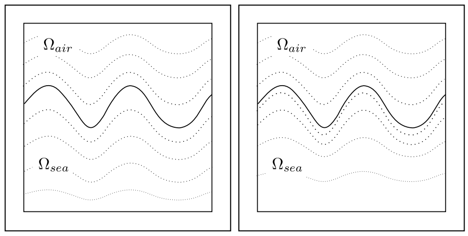
Where x_{int} and y_{int} are the coordinates of the nearest point from $\mathbf{x}$ that belongs to the domain Ω_{int}.

3.2.2 Mesh generation

The geometry of the sea wave is given by a function $y = f_{sea}(x)$ that quantifies the elevation of the sea surface in each point. The derivative and the second derivative of f_{sea} can be computed and the curvilinear elements quite easily form an unstructured mesh of the computational domain [16].

More, as described in figure 6, the horizontal curves of the mesh are obtained using a constinuous distortion from the sea surface ($y = f_{sea}(x)$) to the lower and the upper plane boundaries.

First, a linear distortion was considered (see fig. 6(a)). But, the electrical field exponentially decays in the sea water and this fact leads to important convergence difficulties. Better results were obtained with a distortion that follows the exponential decay in the sea (see fig. 6(b)).



(a) *Linear mesh* (b) *Mesh with exponential decay in the sea*

Figure 6: Unstructure mesh of the domain.

4 Conclusion

The numerical results obtained in this study show that the MoM and the NDGM provide similar results with weak or moderate wind speed. For stronger wind conditions, significant differences can be pointed out.

Finally, it is shown that the NDGM is a valuable approach to estimate the electromagnetic field scattered by a sea surface in a canonical configuration. The NDGM should be an interesting methodology to simulate more complex configurations. Further studies are in developement on this subject.

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