Scattering by a time evolving rough surface based on a curvilinear coordinate approach

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***Abstract*— This paper presents a curvilinear coordinate approach devoted to the estimation of the electric field scattered by a moving perfectly conducting interface.**

I. Introduction

The electromagnetic scattering by rough surfaces is clearly a matter of the most importance in the scope of the remote sensing. And many theoretical models have been developed to describe and quantify these scattering phenomena. Small Perturbation Method (SPM), Kirchhoff Approximation (KA), Two Scale Model (TSM), Small Slope Approximation (SSA) or Integral Equation Method (IEM) can be listed as the most commonly applied approaches. Nevertheless, these approaches must be considered as approximation models and their application domain usually depends on the wavelength-roughness ratio.

For several years, Baudier et al. [1], [2] have developed an exact model based on the curvilinear coordinate method (CCM) that gives the scattering coefficients of a static one-dimensional rough surface illuminated by a plane wave.

In this paper, the time variation of the rough surface is taken into account and a CCM model adapted to time evolving surface is presented.

II. Theoretical models

For the sake of simplicity, the surface illuminated by a plane incident wave is supposed to be a one dimensional (in space domain) perfectly conducting interface, see figure 1. Moreover, this surface is time varying so that the surface height y_s is given by $y_s = f(x,t)$. We assume that the incident wave vector is in the xOy plane and the incident electric field $\vec{E}_i$ is perpendicular $\vec{E}_i = E_i\,\vec{u}_z$. The incident circular frequency and wave vector are respectively denoted ω_i and $\vec{k}$.

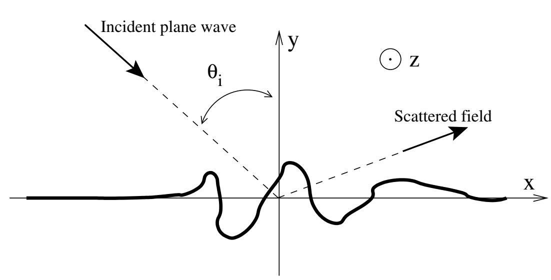

Fig. 1. Geometric configuration

A. CCM with static surface

In the case of a static surface $y_s = f_0(x)$, CCM requires a curvilinear coordinate system (x', y', z') that corresponds to:

$$\begin{cases} x' = x \\ y' = y - f_0(x) \\ z' = z \end{cases} \tag{1}$$

This new coordinate system involves a covariant (resp. contravariant) vector basis $(\vec{e}_{x'}, \vec{e}_{y'}, \vec{e}_{z'})$ (resp. $\left(\vec{e}^{x'}, \vec{e}^{y'}, \vec{e}^{z'}\right)$):

$$\begin{cases} \vec{e}_{x'} = \vec{e}_x + \dot{f}_0(x)\,\vec{e}_y \\ \vec{e}_{y'} = \vec{e}_{y'} \\ \vec{e}_{z'} = \vec{e}_{z'} \end{cases} \quad \begin{cases} \vec{e}^{x'} = \vec{e}_x \\ \vec{e}^{y'} = -\dot{f}_0(x)\,\vec{e}_x + \vec{e}_{y'} \\ \vec{e}^{z'} = \vec{e}_{z'} \end{cases} \tag{2}$$

Due to the nonorthogonality, this new vector basis induces a curvilinear metric tensor $\bar{\bar{G}}^{ij}$ given by:

$$\bar{\bar{G}}^{ij} = \begin{pmatrix} 1 & -\dot{f}_0(x) & 0 \\ -\dot{f}_0(x) & 1 + \dot{f}_0^2(x) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

With these asumptions, the boundary condition at the interface can be expressed in the simple form:

$$\forall x, z, \quad E(x', y' = 0, z') = 0 \tag{4}$$

In curvilinear coordinates, Maxwell equations lead to a linear differential equation system. To solve these differential equations, an expansion in function series can be applied.

Finally, a key point for the CCM approach is that outside the area where the plane interface is modulated, the scattered electric field $\vec{E}_s = E_s\vec{u}_z$ can be written as an outgoing plane wave spectrum, called Rayleigh integral:

$$E_s = \frac{1}{2\pi}\int_{-\infty}^{\infty} w(\alpha)\cdot e^{-i\alpha x}\cdot e^{-i\beta(\alpha)y}\,d\alpha \tag{5}$$

where $\beta(\alpha) = \sqrt{k^2 - \alpha^2}$.

B. Time evolving rough surface

When a time varying surface is considered $y_s = f(x,t)$, space and time coordinates have to be used. The 4-dimension Minkowski spacetime must be considered and the cartesian coordinate system is $\left(x^0 = ct, x^1 = x, x^2 = y, x^3 = z\right)$. In this condition, the four curvilinear coordinates $\left(x^0 = ct'\, x^1 = x', x^2 = y', x^3 = z'\right)$ are in the form:

$$\begin{cases} ct' = ct \\ x' = x \\ y' = y - f(x,t) \\ z' = z \end{cases} \tag{6}$$

These new curvilinear coordinates involve new covariant and contravariant basis vectors.

For the sake of simplicity, it is noteworthy that, in the following, the indexes denoted by a Greek letter range from x^0 to x^3.

1) Covariant basis: Considering the tangent vectors related to each curvilinear coordinate, the curvilinear covariant basis $(\vec{e}_{ct'}, \vec{e}_{x'}, \vec{e}_{y'}, \vec{e}_{z'})$ can be expressed as:

$$\begin{cases} \vec{e}_{ct'} = \vec{e}_{ct} + \frac{1}{c} f_t(x,t)\, \vec{e}_y \\ \vec{e}_{x'} = \vec{e}_x + f_x(x,t)\, \vec{e}_y \\ \vec{e}_{y'} = \vec{e}_y \\ \vec{e}_{z'} = \vec{e}_z \end{cases} \tag{7}$$

where $f_x(x,t)$ (resp. $f_t(x,t)$) is the partial derivative of the function $f(x,t)$ with respect to the first (resp. second) variable.

In the cartesian spacetime coordinate system, the covariant metric tensor is known:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{8}$$

With the change of basis, the covariant metric tensor in curvilinear vector basis becomes:

$$g_{\mu'\nu'} = \begin{pmatrix} 1 - \frac{1}{c^2} f_t^2 & -\frac{1}{c} f_x f_t & -\frac{1}{c} f_t & 0 \\ -\frac{1}{c} f_x f_t & -1 - f_x^2 & -f_x & 0 \\ -\frac{1}{c} f_t & -f_x & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{9}$$

2) Contravariant basis: If the linear equations system (6) is differentiated, the so-obtained equations system leads to the expression of the contravariant basis:

$$\begin{cases} \vec{e}^{ct'} = \vec{e}^{ct} \\ \vec{e}^{x'} = \vec{e}^{x} \\ \vec{e}^{y'} = \vec{e}^{y} - f_x(x,t)\, \vec{e}^{x} - \frac{1}{c} f_t(x,t)\, \vec{e}^{ct} \\ \vec{e}^{z'} = \vec{e}^{z} \end{cases} \tag{10}$$

In the cartesian spacetime coordinate system, the contravariant metric tensor is equal to the covariant one:

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{11}$$

And with the change of basis, the contravariant metric tensor in curvilinear vector basis becomes:

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & -\frac{1}{c} f_t & 0 \\ 0 & -1 & f_x & 0 \\ -\frac{1}{c} f_t & f_x & -1 - f_x^2 + \frac{1}{c^2} f_t^2 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{12}$$

Finally, these analytic expressions of the co- and contravariant vector basis and the co- and contravariant metric tensor enable the determination of the differential operators ($\vec{\nabla}$, $\vec{\nabla}\cdot$, $\vec{\nabla}\times$,...) in the curvilinear geometry.

3) Boundary conditions: In addition to the change from the cartesian to the curvilinear geometry, the moving speed of the interface modify the boundary conditions. Indeed, the boundary conditions at perfectly conducting interface are no longer in the simple form given at the equation (4), but the new boudary conditions must take into account the speed vector $\vec{v}$ at each point of the moving surface. More precisely, the new boundary conditions between two medium are [3]:

$$\vec{n} \times \left(\vec{E}_1 - \vec{E}_2\right) - (\vec{n} \cdot \vec{v}) \left(\vec{B}_1 - \vec{B}_2\right) = 0 \tag{13a}$$

$$\vec{n} \times \left(\vec{H}_1 - \vec{H}_2\right) + (\vec{n} \cdot \vec{v}) \left(\vec{D}_1 - \vec{D}_2\right) = \vec{j}_s \tag{13b}$$

$$\vec{n} \cdot \left(\vec{B}_1 - \vec{B}_2\right) = 0 \tag{13c}$$

$$\vec{n} \cdot \left(\vec{D}_1 - \vec{D}_2\right) = \rho_s \tag{13d}$$

where $\vec{j}_s$ and ρ_s are respectively the surface current density and the surface charge density.

In the case of a perfectly conducting surface, the boundary conditions becomes at $y' = 0$:

$$\vec{n} \times \vec{E} - (\vec{n} \cdot \vec{v})\, \vec{B} = 0 \tag{14a}$$

$$\vec{n} \times \vec{H} + (\vec{n} \cdot \vec{v})\, \vec{D} = \vec{j}_s \tag{14b}$$

$$\vec{n} \cdot \vec{B} = 0 \tag{14c}$$

$$\vec{n} \cdot \vec{D} = \rho_s \tag{14d}$$

4) Maxwell equations: To express the Maxwell equations in the curvilinear geometry, we must introduce the electromagnetic field tensor F and the excitation tensor G. In cartesian coordinate system, these tensors are in the form:

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -cB_z & cB_y \\ -E_y & cB_z & 0 & -cB_x \\ -E_z & -cB_y & cB_x & 0 \end{pmatrix} \quad (15)$$

and

$$G_{\mu\nu} = \begin{pmatrix} 0 & -cD_x & -cD_y & -cD_z \\ cD_x & 0 & -H_z & H_y \\ cD_y & H_z & 0 & -H_x \\ cD_z & -H_y & H_x & 0 \end{pmatrix} \quad (16)$$

Using this formalism, the Maxwell equations can be reduced to:

$$F_{\alpha\beta,\gamma} + F_{\beta\gamma,\alpha} + F_{\gamma\alpha,\beta} = 0 \quad (17a)$$

and

$$G^{\alpha\mu}_{,\alpha} = J^{\mu} \quad (17b)$$

where $\vec{J} = \left(c\rho, \vec{j}\right)$ is the spacetime current density ($\vec{j}$ is the commonly used spatial current density). And, the $,\alpha$ index stands for the partial derivative of the associated tensor with respect to the variable α.

In the present case, we can notice that $\vec{J} = \vec{0}$ outside the moving interface.

In the curvilinear geometry, the Mawxell equations (17) and the boundary conditions (13) lead to a linear equation system. It worth to notice the Mawxell equations (17) provide a great number of linear equations. Nevertheless, a major part of these equations are trivial (antisymmetric tensor) and can be neglected.

5) Incident and scattered field: In the same way as the scattering by a static surface, we considered a plane incident wave so that the incident electrical field $\vec{E}_i$ is given by:

$$\vec{E}_i = E_0 e^{i(\omega_i t - \alpha_i x + \beta_i y)} \vec{e}_z \quad (18)$$

However, in the case of a moving surface, the scattering problem has to be considered as a non harmonic problem and the scattered electric field $\vec{E}_s = E_s \vec{u}_z$ must spread over a Doppler spectrum and the plane wave spectrum corresponding to the scattered field becomes:

$$\vec{E}_s = \frac{1}{2\pi} \iint_{-\infty}^{\infty} w\left(\alpha, \omega_d\right) \cdot e^{-i(\alpha x + \beta(\alpha) y)} \cdot e^{i(\omega_i + \omega_d)t} \, d\alpha \, d\omega_d \quad (19)$$

As a consequence, the linear equation system obtained from the Mawxell equations (17) and the boundary conditions (13) constitutes, in fact, a linear integral equation system.

6) Method of moments: Using a similar methology to the one used in the static surface configuration, the electric and magnetic field estimation is given by a method of moment approach. More precisely, the resolution is based on a ”rectangle/rectangle” method of moment [4], [5].

7) Global algorithm: In a global point of view, using a covariant formalism in Minkowski spacetime and applying a method of moment with curvilinear coordinates lead to a new method to estimate the electromagnetic field scattered by a time evolving rough surface.

In practice, for numerical stability reasons, valuable numerical results are obtained if optimal truncations and explicit limitations are introduced.

Finally, this algorithm provides a quite reliable estimation of the scattering by a moving rough interface as previously described.

III. Numerical results

To evaluate the relevance of our approach, we considered a very simple case of time evolving surface. Indeed, our theoretical methodology has been applied to a rough surface modulated by an oscillating function:

$$y_s = f_0\left(x\right) \cdot \cos\left(\omega_0 \cdot t\right) \quad (20)$$

To reduce the numerical computations, the Doppler spectrum of the scattered field is assumed to be bandlimited with the double of the oscillating function frequency:

$$\vec{E}_s = \frac{1}{2\pi} \int_0^{2\omega_0} \int_{-\infty}^{\infty} w\left(\alpha, \omega_d\right) \cdot e^{-i(\alpha x + \beta(\alpha) y)} \cdot e^{i(\omega_i + \omega_d)t} \, d\alpha \, d\omega_d \quad (21)$$

In these conditions, promising results can be obtained and the Doppler spectrum of the scattered electric field can be clearly pointed out. The Doppler spectrum is estimated with different incidence angles θ_i. The influence of this parameter upon the mean Doppler shift and the length of the Doppler spectrum is highlighted. In the same way, the influence of the circular frequency ω_0 is also discussed.

In any case, the presented results show that the scattering problem raised by moving boundary can not be reduced to a simple Doppler shift. In fact, the physical model must explicitly take into account the continuous spectrum of the scattered field.

The main purpose of the presented study is the evaluation of the theoretical feasibility and the numerical example can be considered as an oversimplify time varying surface. Nevertheless, this study could be of interest for the comprehension of the scattering by natural moving surfaces like ocean surfaces. With this in mind, future works are to be continued.

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