# Model of GPS signal from the ocean based on an electromagnetic scattering theory: a Two Scale Model (TSM) approach

Arnaud Coatanhay and Ali Khenchaf

Laboratoire E3I2- EA 3876,

Ecole Nationale Supérieure des Ingénieurs des Etudes et Techniques d'Armement

29806 Brest Cedex 09, France

Email: arnaud.coatanhay@ensieta.fr

Telephone: (33) 2 98 34 88 09

Fax: (33) 2 98 34 87 50

*Abstract*— In this paper, we present a simulation of the sea surface scattering for GPS signal reception in a maritime context. Our computation is based on a fully bistatic Two Scale Model (TSM). It is noteworthy that the geometric description of the sea surface is given by a realistic spectrum (Elfouhaily spectrum) and a slope probability density function(Cox-Munk distribution).

## I. INTRODUCTION

In maritime environments, the GPS system constitutes one of the most obvious way for ship positioning, and the simulation of GPS signals in a maritime environment to evaluate receiver robustness is of a great importance. More, in already published studies [1], GPS signal scattered by the sea surface has been proved to be a very promising source of information for wind speed estimation. So, a reliable simulation of the GPS signal based on the physical properties of the sea has become very necessary.

In the literature, an electromagnetic simulation of the GPS signal based on Kirchhoff Approximation (KA) has already been presented [2]. Nevertheless, the Kirchhoff Approximation (KA) only takes into account scattered signals close to the specular reflection, and the diffuse reflections are almost neglected.

For the sake of precision, we present the simulation of GPS signal based on a Two Scale Model (TSM) [3]. Our approach takes into account the diffuse reflections and the cross polarization components. Different numerical evaluations are considered with wind speed and wind direction as parameters. With regard to the sea surface description, we applied the Cox-Munk slope probability density function and the Elfouhaily unified spectrum [4].

This computation can provide a simulation of the GPS signals in maritime environments as a function of various atmospheric parameters.

## II. PHYSICAL MODELS

### A. Sea surface description

Obviously, a reliable simulation of GPS signals above the sea involves a realistic description of the sea surface roughness. And, this description must be a function of wind speed and direction. The most standard ways to characterize the roughness are the spectrum and slope probability density function.

*1) Sea slope distribution:* The sea surface, considered as a random process, could be characterized by the slope probability density $P(Z_x, Z_y)$. Although basic forms of probability density functions (gaussian expressions for example) could be used for a coarse description, a more sophisticated model is required for realistic simulations.

Based on the analysis of sun glitter photographs, Cox and Munk [3], [4], [5] generated a more reliable semi-empirical slope distribution law.

*2) Sea spectrum:* The second standard way to describe the roughness of the sea surface is to determine the sea surface spectrum $S(K, \phi)$, considering the sea surface as a random, ergodic and stationary process. In scientific literature [6], [7], [8], [9], many papers provides fully detailed description of various sea spectra, see Pierson and Moskovitz studies [10], [11] for instance. In this paper, we considered the Elfouhaily spectrum [12], called unified spectrum, that is very consistent with actual observations and presents no discontinuity at gravity and wind driven waves.

The sea spectrum is in the form:

$$S(K, \phi) = M(K)f(K, \phi) \qquad (1)$$

where $M(K)$ represents the isotropic part of the spectrum modulated by the angular function $f(K, \phi)$. $K$ and $\phi$ are respectively the spatial wave number and the wind direction.

The both surface representations (slope probability function and sea spectrum) are a key feature when estimating the electromagnetic sea surface scattering of a GPS signal by the sea surface.

### B. Electromagnetic scattering

A plane wave impinging a rough surface is scattered in any direction. Indeed, the sea surface "reflection" of the incident

wave, coming from the satellite, must be considered as a fully bistatic configuration, see figure (1).
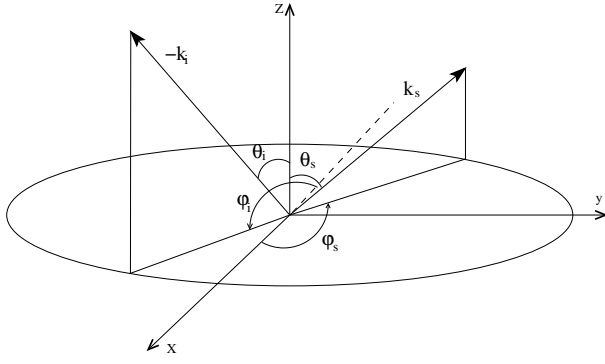


Fig. 1. Geometric configuration

For a given $E^i$ incident wave, the $S$ scattering matrix provides the scattered polarization and amplitude of the $E^s$ scattered wave:

$$E^s = \begin{bmatrix} E^s_{v_r} \\ E^s_{h_r} \end{bmatrix} = \begin{bmatrix} S_{v_r v} & S_{v_r h} \\ S_{h_r v} & S_{h_r h} \end{bmatrix} \begin{bmatrix} E^i_v \\ E^i_h \end{bmatrix} \quad (2)$$
$$= [S]E^i$$

Many approaches were developed to evaluate electromagnetic this scattering matrix. In this paper, cited approaches are: Geometrical optics or physical optics methods, called Kirchhoff Approximations (KA) [13], Small Perturbation Method (SPM) [14], [15] and Two-Scale Model (TSM) [16], [17].

*1) Kirchhoff Approximation (KA):* In few words, KA approach [13] assumes the sea surface can be approximated by a tangent plane at each point of the surface, see figure (2).
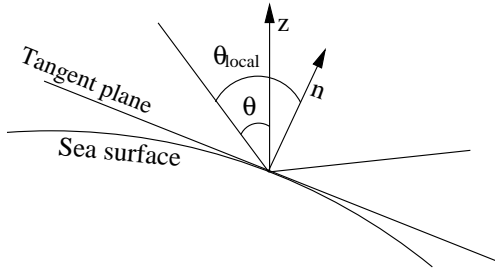


Fig. 2. Geometric optic approximation

The scattered waves is in the form:

$$\vec{E}^s = K\vec{n}_s \wedge \int \{\vec{n} \wedge \vec{E} - \eta \vec{n}_s \wedge (\vec{n} \wedge \vec{H})\} e^{jk\vec{r}.(\vec{n}_s - \vec{n}_i)} ds \quad (3)$$

where $\vec{n}_s$ is the unit vector in the scattered direction, $\vec{n}$ is the unit vector normal to the surface, $\eta = \frac{\mu}{\epsilon}$ is the intrinsic impedance of the medium, $\vec{E}$ is the total electric field and $\vec{H}$ is the magnetic field. Using severals assumptions, the scattered waves can be obtained from the slope probability density function of the sea surface (Cox-Munk distribution).

The KA approach is only valid for a surface with an important horizontal roughness scale and average curvature radius compared to the electromagnetic wavelength. This approach is well adapted to the scattering by the gravity waves and to the computation of the specular component.

*2) Small-Perturbation Method (SPM):* On the contrary, the SPM approach is more adapted to small roughness scales and small sea amplitude. That is the case for wind driven waves. The SPM model fits the experimental data when the phase difference due to height variation (Rayleigh criterion) is much smaller than $2\pi$, and the slope is much smaller than unity. The computation of the SPM is based on a spectral description of the sea surface (Elfouhaily spectrum). Actually, the first order SPM approach provides an accurate estimation of the diffuse component (Bragg scattering) of the electromagnetic scattered wave.

*3) Two-Scale Model (TSM):* The purpose of the TSM approaches is to take advantage of the both KA and SPM validity domains and to manage the both roughness scales (gravity waves and wind driven waves). Quite recently, Khenchaf developed a robust two-scale model [16], [17] that can be applied to determine the direct or cross polarization coefficients in fully bistatic configurations. The figure (3) schematically represents the both roughness scales of the sea.
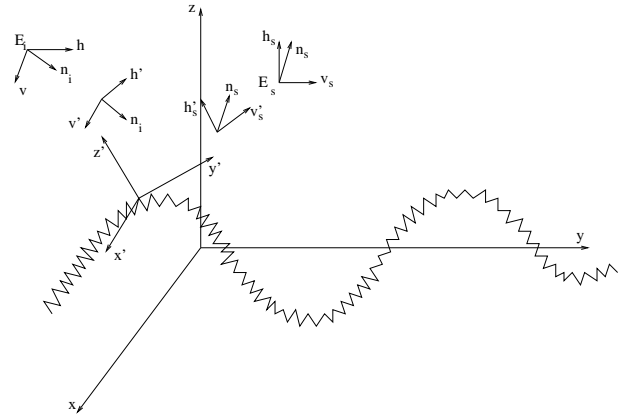


Fig. 3. surface geometry

For small scales, the main point of this approach is to to considered the tangent plane related to the gravity waves as a local reference. With this tilting process, the scattering by the wind driven sea waves is estimated with the SPM approach (Elfouhaily spectrum) weighted with probability of the tangent plane slope (Cox-Munk distribution). The specular contribution (computed with KA) added with the integration of these local contributions provides a very reliable Two Scale Model.

Finally, the TSM approach based on sea surface description (slope probability density function or sea spectrum) allows to estimate the scattering coefficient due to an elementary sea surface, see figure (4), as a function wind direction and wind speed.
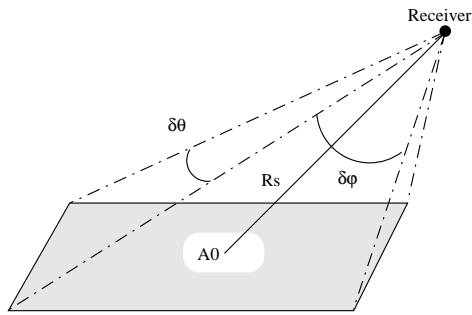
Fig. 4. Elementary sea surface

## C. GPS signal simulation

The simulation of the GPS signal consists to add the contribution of each elementary sea surface. More, each elementary contribution is associated with a random phase (between 0 and $2\pi$) so that the sum is incoherent.

Nevertheless, this sum must take into account the delay related to each ray (elementary contribution). Therefore, we must add the contributions with the same delay. In the present case, these contributions correspond to sea surfaces in the area limited by intersection of two Fresnel ellipsoids, see figure (5). Then, these annulus zones, between two iso-range
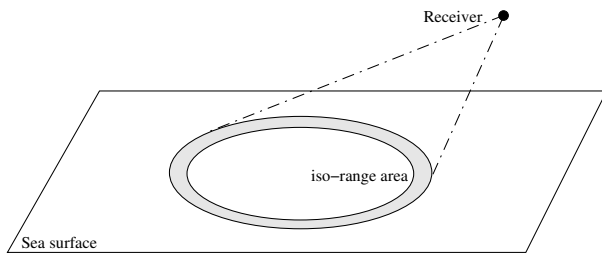


Fig. 5. Elementary iso-range area

lines, are divided into a great number of angular sectors to obtain elementary surfaces.

To compute the scattering coefficients using the TSM approach, the GPS signal whose carrier wave is circularly polarized, is split into two linearly polarized waves (horizontal and vertical components).

## III. NUMERICAL RESULTS

The figure (6) shows a simulation example of the impulse response of the GPS signal for receiver at 50 meters above the sea with an elevation angle (satellite direction) equal to $45°$. The amplitude is related to the amplitude of the direct line of sight signal (no reflection by the sea), and the delay is related the delay between the direct signal and the specular signal.

The figure (7) illustrates the distribution of the power density (impulse response) received by an observer situated at 5000 m above the sea with a strong wind (Beaufort scale coefficient = 10). The elevation angle is equal to $45°$. For a quite low coefficient on the Beaufort scale, the sea scattering

can be reduced to a quasi specular reflection. But, when the coefficient is higher, the diffuse component is important and the average delay of the scattered signals grows.

## IV. CONCLUSION

In this paper, a simulation of the GPS signals based on a bistatic TSM electromagnetic model is presented. More, this simulation takes into account realistic descriptions of the sea surfaces (Elfouhaily sea spectrum and Cox-Munk sea slope probability density function) that depends on the wind speed and direction.

These numerical results obtained with a TSM approach constitutes a first step. In next studies, comparisons with more classical approaches will be presented.

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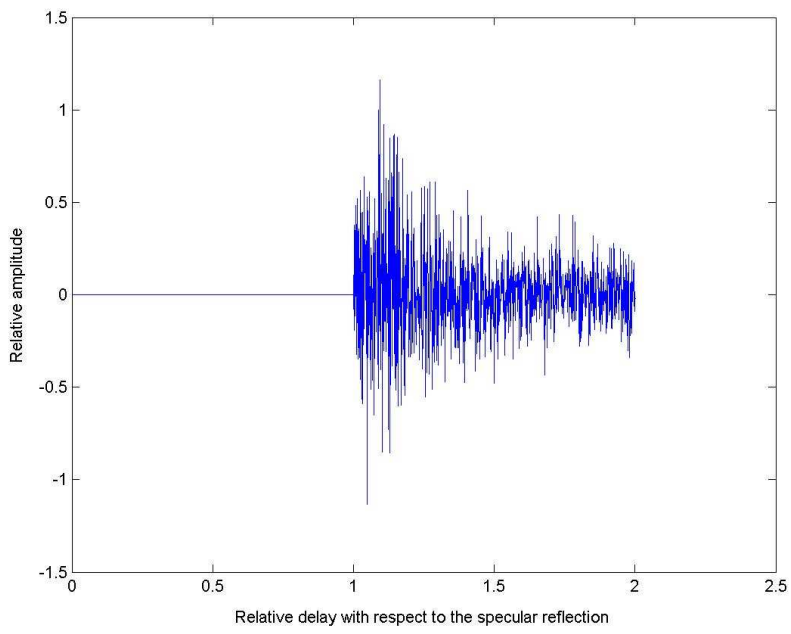
Fig. 6. Numerical simulation of a GPS signal (impulse response) received above sea surface (Beaufort scale coefficient = 6.5)
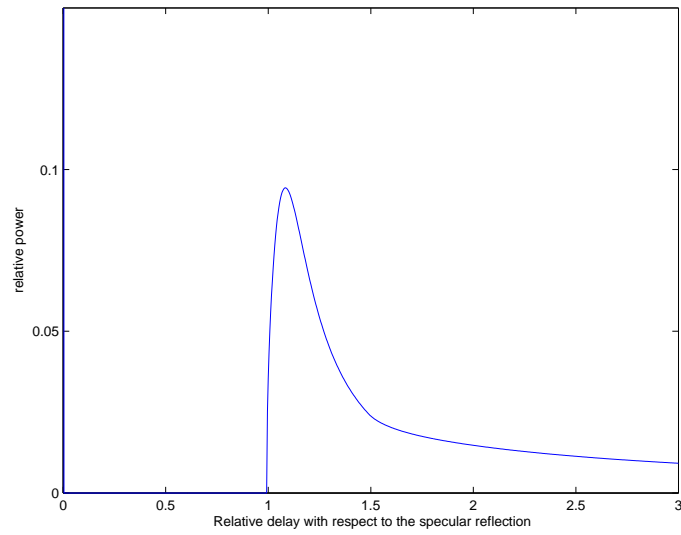


Fig. 7. Numerical simulation of a GPS power density received above sea surface (Beaufort scale coefficient = 10)