Regularization of Laplace transform inversion for subsurface conductivity and permitivity profile estimation using GPR signals.

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***Abstract*— This paper deals with the regularization problem raised by the inversion of GPR radargram. The determination of subsurface conductivity and permitivity profiles leads to Laplace transform inversions for real data. A very recent method to solve this severely ill-posed problem is presented and evaluated using a standard 2D FDTD numerical GPR model for multilayered media.**

***Index Terms*— GPR, Laplace transform inversion, Inverse problem, Layered media.**

I. Introduction

In this paper, the subsurface is made of horizontal layered media, so that the conductivity σ and the pemittivity ε only depend on the depth (z-axis), and the plane (x, y) is supposed to be parallel to the Earth's surface. Assuming the GPR device could be modeled by a current source $\vec{j}$, the ground penetrating problem is mathematically expressed by the Maxwell's equations:

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}, \quad \nabla \times \vec{H} = \varepsilon (z) \frac{\partial \vec{E}}{\partial t} + \sigma (z) \vec{E} + \vec{j} \quad (1)$$

where $\vec{E}$, $\vec{H}$, μ are electrical and magnetic fields and the magnetic permeability respectively.

If the current source is a straight wire $\vec{j} = f(t)\,\delta(x)\,\delta(z - z_0)\,\vec{e}_y$ or a loop of current $\vec{j} = f(t)\,\delta(r - r_0)\,\delta(z - z_0)\,\vec{e}_\Phi$, the GPR problem can be reduced to an ordinary differential equation, see [1], [2]:

$$u'' - \lambda^2 u + \omega^2 A^2(z)\, u + i\omega B(z)\, u = -\delta(z - z_0) \quad \text{(2a)}$$

with

$$u(\pm\infty, \lambda, \omega) = 0 \quad \text{(2b)}$$

where $A = \varepsilon\mu$, $B = \sigma\mu$ and u depends on Fourier transforms and/or Hankel-Bessel transforms of the $\vec{E}(x, y, z, t)$ and $f(t)$ functions.

Then, it can be proved that the Laplace transform of $A^2(z)$ and $B^2(z)$ functions can be determined for this physical problem. In this way, the subsurface conductivity and permitivity profile estimation with GPR boils down to the Laplace transform inversion problem.

II. Laplace transform inversion

Most of numerical algorithms dedicated to the numerical inversion of Laplace transforms (Zakian or FFT algorithms for example), considers Laplace transform as a complex-variable function, even though, in the present case, the Laplace transforms are only known on real axis. Moreover, the real Laplace inversion constitues a highly ill-posed problem.

Several years ago, A. G. Ramm proposed a theoretical solution for the real Laplace inversion, see [3], but he did not manage the regularization of the ill-posedness. More recently, L. D'Amore and A. Murli, see [4], described a regularized algorithm for the real data Laplace inversion.

This method is based on the representation of the Laplace transform in terms of rational function series. This representation and coefficient identification is obtained using a least square minimization and regularized with a Tikhonov process. In few words, the algorithm express the inverse Laplace transform in the form of a modified Fourier series.

III. Numerical evaluation

Finally, the conductivity and permittivity profiles can be numerically estimated with a global algorithm. The accuracy and stability is evaluated for computed data coming from a 2D FDTD simulator for ground radar modelling.

Numerical simulations are provided for different layered media and with several elementary perturbations. The influence of the Tikhonov parameter for the regularization is pointed out.

References

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