Optimized GPR signal deconvolution using an adaptative conjugate gradient method

A.Coatanhay
Laboratoire en Extraction et Exploitation d'Infomation en Environement Incertain (E3I2)
ENSIETA, 29806 Brest cedex 9, France. e-mail: coatanar@ensieta.fr

***Abstract* -This paper presents a signal processing method to improve the identification of interface between different layered media, using a deconvolution process. Our methological approach is based on an inverse problem algorithm, called conjugate gradient method. The emitted pulse is assumed to be known, and the convolution is considered as a linear operator that must be inversed. More, in this paper, we take into account the distortion of the signal and the noise of the recorded signal.**

I. INTRODUCTION

This paper presents a signal processing method with a good compational tactability to improve the identification of interfaces between different layered media, from a Ground Penetrating Radar (GPR) recording. Our methological approach is based on an inverse problem regularization algorithm called conjugate gradient method.

The backscattering of a normal incidence radar wave by a horizontally stratified media can be expressed as the convolution between two basic components: the reflectivity sequence s and the emitted pulse e.

$$r = s * e \qquad (1)$$

The recieved r signal is actually recorded and the e pulse is supposed to be known. On the opposite, the reflectivity sequence that is a succession of dirac impulses pointing out interfaces between homogeneous layers, has to be estimated.

II. CONJUGATE GRADIENT METHOD

From a mathematical point of view, the $K: f \to f * e$ is a linear operator, and the deconvolution problem bolds down to the estimation of the inverse operator K^{-1} . It is noteworthy that the adjoint operator of K is easily determined: $K^*: f \to f * \hat{e}$ with $\hat{e}(t) = e(-t)$. In this case, the iterative conjugate gradient method, see [1,2], seems to be a consistent approach to estimate the K^{-1} operator. In a few words, the conjugate gradient method, for a $K \cdot x = y$ inverse problem, consists in estimating the optimal inverse solution by constructing a sequence of subspaces $D_0, D_1, \cdots, D_n$ with the property: $D_0 \subset D_1 \subset \cdots \subset D_n$. At each iteration, we will find a solution x_n which minimizes the quadratic error $\| K \cdot x_n - y \|^2$. The initial vector x_0 is set to zero, and the successive subspaces, called Krylov subspaces, are in the form:

$$D_n = span\left\{ p^0, K^* K \cdot p^0, \cdots, \left(K^* K \right)^n \cdot p^0 \right\} \qquad (2)$$

with $p^0 = -K^* \cdot y$. For a finite dimension m problem, this algorithm stops at the mth iteration, and corresponds to a standard least square optimization approach. For an infinite inverse, this process asymptotically converge towards an optimal solution.

In reality, the r signal is recorded from actual condition mesures, and does not strictly correspond to the theoretical linear model. In these conditions, the iterative inversion process appears as a divergent method. In fact, the conjugate gradient method gives a good estimation of the K^{-1} operator if an adapted stopping criterion is chosen. The problem is the appropriate number of iteration must be determined from the level of pertubation (electronic noise, undefined interaction with the layers, non-linearity etc...) in the r signal, see [3], and this level is hardly ever accurately estimated, in practice. More, the signal to noise ratio depends on which part of the r signal is studied. In any case, the signal to noise ratio can not be considered as a constant. To get round these raised problems, we computed, for a great number of terms, the successive estimations of the reflectivity sequence from each iteration of the conjugate gradient method. Then, using a sliding window, we locally select the optimized reflectivity sequence estimation from a given criterion. In fact, the reflectivity sequence estimation must be close to a dirac impulse sequence, and our optimisation is based on an adapted normalized entropy, see [4] :

$$criterion = \frac{1}{N \cdot \log N} \sum_{i=t+1}^{t+N} q_i \cdot \log q_i \qquad (3)$$

with $q_i = \dfrac{r(i)^2}{\sum r(i)^2 / N}$.

We assumed that the optimal solution for the normalized entropy criterion corresponds to an approximation of a dirac impulse sequence. So that, we try to estimate the

0-7803-7537-8/02/$17.00 (C) 2002 IEEE

real reflectivity sequence s. The problem is this estimation comes from a numerical algorithm that takes into account a very basic model (linear convolution) for the underground propagation of the electromagnetic pulse emitted by the GPR.

III. DISTORTION MODEL

In reality, our algorithm is more complex than previously described because we considered the distortion of the emitted pulse e due to the propagation in the ground. As a physical propagation model, the electric wavefield is supposed to be caused by an infinite horizontal line sources (approximation of dipole antennas). Through our choice of horizontally layered soil and line sources, the problem is reduced to two dimensions (x,z). Letting these assumptions, Schoolmeesters, see [5], proved that the propagation in an homogeneous medium is:

$$E(x,z,t)=H(t-T)\frac{\mu}{2\pi}\left[\int_{t'=T}^{t}\frac{I'(t-t')e^{-(\sigma/2\varepsilon)t'}}{\sqrt{(t')^2-T^2}} + \frac{\sigma}{2\varepsilon}\int_{\tau=T}^{t'}I'(t-t')e^{-(\sigma/2\varepsilon)t'}\times \int_{\tau=T}^{t'}\frac{\tau I^{(1)}\left(\frac{\sigma}{2\varepsilon}\sqrt{(t')^2-\tau^2}\right)}{\sqrt{(\tau)^2-T^2}\sqrt{(t')^2-\tau^2}}d\tau dt'\right] \quad (4)$$

where I' is the time-derivative of the source wavelet, T is the arrival time of the pulse, $H(t)$ is a step-function, $I^{(1)}$ is the modified Bessel function of the first kind and order one , and t' and τ are integration variables. So the variations of the emitted pulse shape mainly depend on the arrival time T and the σ/ε ratio that can be estimated to fit the recorded signal. In fact, we determined these parameters in adjusting the emitted pulse shape to obtain the optimal deconvolution process versus the normalized entropy criterion. In consequence, the criterion is needed to locally optimize the number of iteration in the conjugate gradient process and the parameters of the propagation (T and σ/ε).

It is noteworthy that the propagation model is not complex enough to estimate the physical parameters of the different layered media, but it can give a plausible distortion of the emitted pulse. Here, the physical model characterise the propagation in an homogeneous medium. To accurately determine the propagation in different layered media, horizontal boundary conditions must be analyzed at each layer interface, and this approach leads to a far more complex recursion formula. More, these recursion formula raise a difficult inverse problem. So, for us, the only purpose of our simplified propagation model is to improve the estimation of the deconvolution algorithm. And the obtained T and σ/ε parameters can not be used for a direct physical interpretation.

IV. EXPERIMENTAL RESULTS

Finally, our methodology was tested with experimental GPR signals, recorded on a geophysic reference site made of different known materials. First, there is a layer of sand (thickness: 0.80 m), then little gravels (0.60 m) and more bigger gravels (0.60 m), another layer of snad (1.30 m), and the deepest layer is made of limon (0.60m). The frequency of the emitted electromagnetic pulse is about 400 MHz. Due to the computational tactability of our algorithm, a signal deconvolution process can be operated for each position of the antenna (each trace). So, we can easily obtain a deconvoluted radargram of the geophysic site. Our algorithm is not a very sophisticated deconvolution algorithm, but can easily improvre the radargram legibility.

ACKNOWLEDGMENT

The author would like to express his gratitude to the Laboratoire Central des Ponts et Chaussees (LCPC), Bouguenais, France, that provides experimental GPR recordings for this study.

REFERENCES

[1] H. W. Heinz, M. Hanke and A. Neubauer, Regularization of Inverse Problems, Kluwer Academic Publishers, London, 1996.

[2] J. R. Shewchuk, An Introduction to the Conjugate Gradient Method Without the Agonizing Pain, Technical Report, Shool of Computer Science, Pittsburgh, 1994.

[3] R. Plato, The conjugate gradient method for linear ill-posed problems with operator perturbations, *Numer. Algorithms*, vol. 20, n. 1, 1994, pp. 1-22.

[4] M. D. Sacchi, D. R. Velis and A. H. Cominguez, Minimum entropy deconvolution with frequency-domain constraints, *Geophysics*, vol. 59, n. 6, 1994, pp. 938-945.

[5] J. W. Schoolmeesters and E. C. Slob,Effect of the conductivity/permittivity ratio on the dispersion and attenuation of radar signals}, *The First Latin American Geophysical Conference and Exposition*, Rio de Janeiro, Brazil, 1995.