EPJ manuscript No.
(will be inserted by the editor)

Numerical modeling of electromagnetic waves scattering from 2D coastal breaking sea waves

R. Khairi[1 a], A. Coatanhay[1], A. Khenchaf[1], and Y. M Scolan[2]

[1] Lab-STICC UMR CNRS 6285, ENSTA-Bretagne,
[2] LBMS-DFMS EA 4325, ENSTA-Bretagne,
2, rue Francois Verny, 29806 Brest cedex 9, France

Received: date / Revised version: date

Abstract. The aim of this work is modeling the interaction of L-band electromagnetic waves with coastal breaking sea waves. The breaking waves profiles are generated using the Desingularized Technique and the electromagnetic waves scattering are computed using the High-Order Method of Moments (HO-MoM) with a Non Uniform Rational Basis Spline (NURBS) geometry. Our study mainly focuses upon the electromagnetic waves behavior in the crest and the cavity of breaking sea waves.

1 Introduction

In 2008, ENSTA-Bretagne, Telecom-Bretagne and IFREMER launched the Marine Opportunity Passive Systems (MOPS) project [1]. The objective of this project is obtaining the oceanographic information using electromagnetic waves scattering from coastal breaking sea waves. The sources of the electromagnetic waves in this case are the GNSS satellites (L-band passives electromagnetic waves sources) and the observation points are placed a dozen of meters above the sea surface (near field configuration) on the coast.

One of the MOPS project challenge is modeling precisely the interaction of L-band electromagnetic waves with the breaking sea waves. The modeling involves two research domains: "hydrodynamic" to generate the breaking wave profiles, and "electromagnetic" to compute the electromagnetic waves scattering.

The hydrodynamic theory shows that the presence and the evolution in time of the breaking sea wave, depends on the sea floor slope. To model the breaking waves, different numerical approaches can be like Finite Element Method, Boundary Integral Method [2] or the long-tank model [3]. In this work (see the second section), we generate the breaking wave profiles with the FSID (Free Area IDentification) code. A more relevant numerical solution based on a Desingularized Technique provides a robust and reliable simulation of highly non-linear waves [4] in a shallow water context.

On the the other other hand, to model the electromagnetic waves scattering, we apply a boundary element method, meaning that we compute in the first time the currents generated by incident waves and than using these currents, we determine the waves scattering every where in the space. Unfortunately, it is well known that the standard boundary element approaches do not provide a reliable estimation for the electromagnetic field scattered by a breaking wave [5,6].

Indeed, the breaking sea waves profiles have a strong positive and negative curvature and the standard MoM approaches raise convergence problems [7]. To compute the surface currents induced by these profiles, we propose to use the High-Order Method of Moments (HO-MoM) combined with Non Uniform Rational Basis Spline (NURBS) meshing technique. This technique will be presented in the third section. Finally in the last section, we show some simulation results. We focus our results into the behavior of the waves in the crest and the cavity of breaking sea waves.

2 Breaking sea waves modeling

Sea wave is a complex physical phenomenon that involves nonlinear physics modelling. In many remote sensing applications, the sea surface is considered as a random physical system and its representation is given in terms of sea spectra: Pierson-Moskowitz, Elfouhaily etc. These spectra give a statistical information of the sea surface profile for a given location on ocean. However, the spectrum representation cannot give the information about the sea wave dynamics in general and the coastal breaking waves in particular. The hydrodynamic theory is required to respond to this problem.

The fundamental characteristic of coastal breaking waves is that their dynamics (and therefore their geometry) depend on the variation of the sea depth [8]. When the waves approach the coast, the depth decreases rapidly and the waves reach their limit of stability and break, even for a

[a] e-mail: *refzul.khairi@ensta-bretagne.fr*

small wind speed. Thus, the fluid mechanics imposes that the structure of the waves mainly depends on the slope of the coast bathymetry.

There are different classifications of breaking waves. These classifications are based on the form of the waves crest at their critical steepness. Galvin classified them in mainly two types: spilling and plunging [9]. Spilling breaking waves in one hand, occur when the ocean floor has a gradual slope. They break for a longer time than other waves, and create a relatively gentle waves. Plunging waves in other hand, occur when the ocean floor is steep or has sud- den depth changes, such as a reef or a sandbar. The crest of the waves becomes much steeper than a spilling wave, becomes vertical, then overturns and hits the trough of the next wave, releasing most of its energy at once in a relatively violent impact. Figure (1) illustrate these two breaking waves types.

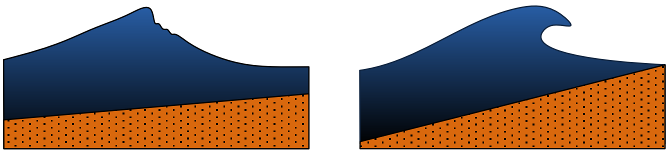

Fig. 1: Breaking waves types (i) Spilling (ii) Plunging

To model the movement of the breaking waves, the hydrodynamic theory is required. This theory is based on three fundamental equations: mass balance or continuity equation, momentum balance or Navier-Stokes equation and energy balance. To obtain a solvable equation, these equations are completed by standard simplification hypotheses. In sea case, we can take the hypotheses that the fluid is ideal, incompressible and irrotational. These simplification derive two principle equations in the breaking waves study: the Laplace equation

$$\nabla^2 \psi = 0 \tag{1}$$

and the Bernouilli equation

$$\frac{\partial \psi}{\partial t} + \frac{1}{2} |\nabla \psi|^2 + \frac{P}{\rho} + gz = 0 \tag{2}$$

where ψ is the scalar vector of the fluid, ρ is the fluid density, P is the pressure, and g is the gravity.

Applying the kinematic and dynamic boundary conditions in free surface and the floor of the waves, the equations (1 & 2) can than be solved. The analytical solution for these equations are given by the theory of Airy, Stokes and Conoidal. However, these theories are limited to model the non-breaking waves.

The modeling and numerical simulation of a breaking wave is still a challenge when dealing with a realistic three-dimensional see state. The Computational Fluid Dynamics do not provide yet efficient, robust and accurate tools. With no doubt Potential Theory is the best framework in which the limit of stability of gravity waves can be reproduced. Three-dimensional configurations are still a challenge for industrial purposes, however two-dimensional con-

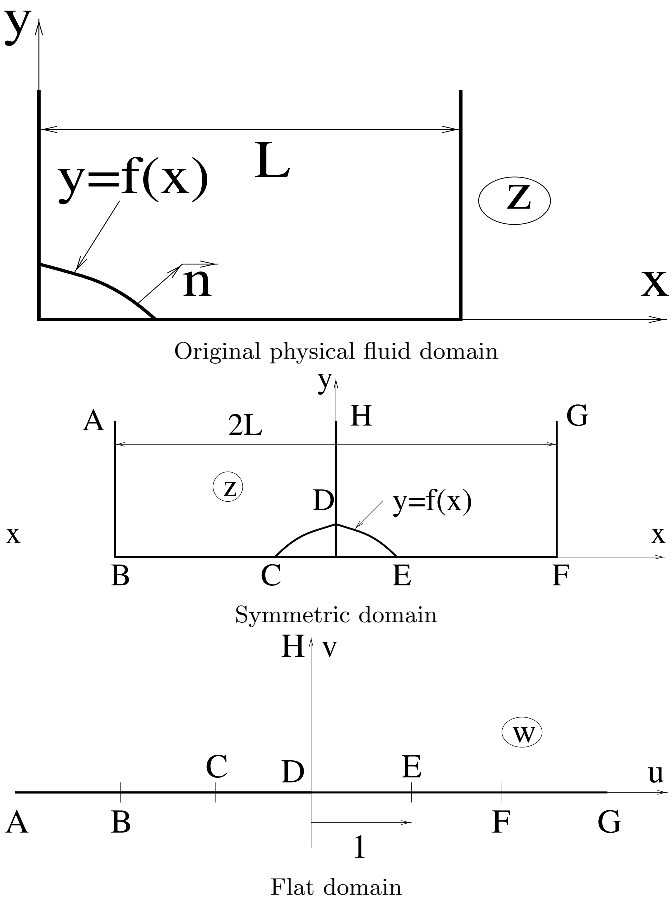

Fig. 2: Holomorphic transformation of the fluid domain

figurations can be routinely simulated. Among the numerous numerical tools, the so-called desingularized technique is known to be very robust and does not suffer from drawbacks (like regridding and smoothing) of the standard Boundary Element Methods. For the present applications, where both spilling breakers and overturning crest are modeled in combination with the influence of the bathymetry, there are even optimized techniques which make possible to implicitly account for solid boundaries. By using conformal mappings of the fluid domain, the time varying problem can be formulated in terms of tracking markers on the free surface only. Those markers carry the information about the presence of impermeable frontiers (sea bottom, walls,...). In practice a succession of conformal transformations is used to turn the original physical fluid domain into a quarter (or half) space. The sketch below sums up these transformations.

From the physical z-plane (plus its symmetric part with respect to the left vertical wall), we 'flatten' the two vertical walls (AB and FG) by using an integrable Schwartz-Christofel transformation. Then the symmetric domain with respect to the horizontal axis is introduced. The local bathymetry is now a closed contour $CDE+sym.$ which is transformed by using successively a Karman-Trefftz transformation and a Theodorsen-Garrick transformation; we arrive at a unit circle which is finally turns into a flat plate.

For the present two-dimensional potential flow, the Green function is of *log* type (Rankine source). This singularity and its three images with respect to the horizontal and vertical axes verify the initial boundary value problem except the boundary conditions on the free surface. The total velocity potential ϕ is thus expressed as a finite sum of these singularities denoted G

$$\phi(x,y,t) = \sum_{j=1}^{N} q_j(t) G(x,y,X_j(t),Y_j(t)) \tag{3}$$

where (X_j, Y_j) are the source location and q_j is the strength of source j. The singularities are located outside the fluid domain at a short distance from the actual free surface; the technique is thus said desingularized. The velocity potential is updated by solving the dynamic (isobar surface) and kinematic (material surface) boundary conditions written in Lagrangian form

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{1}{2}\left(\boldsymbol{\nabla}\phi\right)^2 - gY, \qquad \frac{\mathrm{d}X}{\mathrm{d}t} = \phi_{,x}, \qquad \frac{\mathrm{d}Y}{\mathrm{d}t} = \phi_{,y} \tag{4}$$

where g is the acceleration of gravity and (X, Y) are Lagrangian coordinates of a marker which moves with the fluid velocity $\boldsymbol{\nabla}\phi = (\phi_{,x}, \phi_{,y})$. These algorithms are implemented in the code FSID (Free Surface IDentification), they are fully described in [4]. Here we use that code to produce realistic overturning crest and its dynamics as illustrated in figure (3).

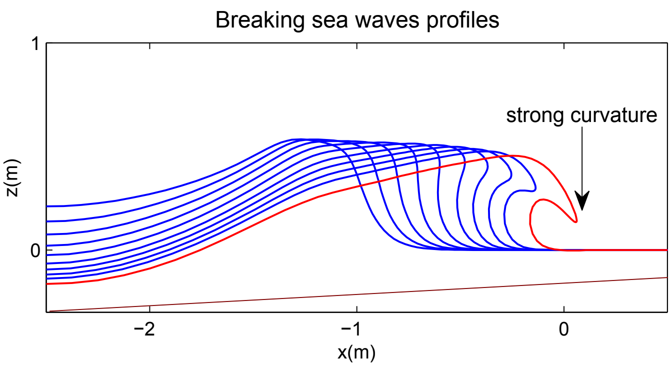

Fig. 3: Breaking waves profiles

3 Electromagnetic waves scattering modeling

Once the breaking waves profiles are determined, we can compute the electromagnetic waves scattering from these profiles. The scattering problem can be seen as a special case of the radiation problems in which the sources (currents) are generated by the incident waves. Thus, the first step to solve the scattering problem is to find the currents. Once the currents found, we can easily use the radiation equation to compute the field every where in the space.

At the boundary of the media, we can make a link between the incident waves and the surface currents. The boundary integral equations in this case are given respectively by the Electric Field Integral Equation (EFIE) and the Magnetic Field Integral Equation (MFIE):

$$\hat{n} \times \overline{E}^{inc}(\overline{r}) = -\hat{n} \times \int \left\{ i\omega\mu \overline{J}_s G + \frac{1}{i\omega\epsilon}\left(\nabla' \cdot \overline{J}_s\right) \nabla' G \right\} d\overline{r}' \tag{5}$$

$$\hat{n} \times \overline{H}^{inc}(\overline{r}) = \frac{\overline{J}_s(\overline{r})}{2} - \hat{n} \times \int \overline{J}_s \times \nabla' G d\overline{r}' \tag{6}$$

where $\overline{E}^{inc}$ and $\overline{H}^{inc}$ are the electric and magnetic incident waves, $\overline{J}_s$ are the surface currents, G are the Green function, μ is the magnetic permeability, ϵ is the electric permittivity, $\hat{n}$ are the normal vector to the surface, $\overline{r}$ and $\overline{r}'$ are the observation and source points.

Except for several canonical geometries like a cylinder or the sphere, the exact analytical solution does not exist for the equation (5 and 6). For complex geometries, some simplification (asymptotic methods) can be made to approximate the exact solution. These methods are valid only for some type of surfaces and scattering mechanisms and not accurate in general. However, they still in great interest thanks to their small computation time. Another possibility is to use the numerical method. Among them we can cite the Method of Moments.

Method of Moments (MoM) is a standard numerical technique to convert the integral equations in to matrix linear systems. The procedure for applying the MoM involves three steps [10]:

- Construction of the integral equations to present the systems
- Discretization of the integral equations in to a matrix linear equation using the basis and testing functions
- Solving the matrix equations to obtain the unknown coefficients

The electromagnetic waves scattering system are presented by the EFIE and MFIE. In general form, we can write:

$$\mathbf{S} = \mathcal{L}\mathbf{f} \tag{7}$$

where $\mathbf{S}$ are the source functions (incident waves), $\mathcal{L}$ are the integral operators and $\mathbf{f}$ are the unknown functions (currents).

To discretize the equation (7), the unknown functions $\mathbf{f}$ are approached by linear combination of the base functions $\mathbb{B}_b,\ b = 1, 2, \cdots, B$.

$$\mathbf{f} \approx \sum_{b=1}^{B} I_b \mathbb{B}_b \tag{8}$$

where I_b are the unknown coefficients to be found. Into two side of the equations, we introduce the testing functions $\mathbb{T}_a,\ a = 1, 2, \cdots, A$:

$$\langle \mathbb{T}_a, \mathbf{S} \rangle = \sum_{b=1}^{B} I_b \langle \mathbb{T}_a, \mathcal{L}\mathbb{B}_b \rangle \tag{9}$$

Than we can make the linear matrix equation (9) in form

$$\mathbf{V} = \overline{\mathbf{Z}}\mathbf{I} \tag{10}$$

The basis and testing function have to be chosen in the same space as the unknown function (Galerkin approach). In classical technique (Classic-MoM), one use the pulse function if the continuity is not required and triangle (2D) or RWG/rooftop (3D) in other case. These basis functions involve the discretization step in the order of $\lambda/10$. To increase this step, which means decrease the unknown coefficient number, we can use the high-order polynomials as the basis (High-Order Method of Moments (HO-MoM)). Other advantage is that the high-order polynomials basis function can improve the computation convergence.

The choice of the basis polynomials is based on two criterion: 1) they must allow the imposition of the continuity 2) they have to be orthogonal to avoid the bad matrix condition. The Bernstein polynomials for example, fill the first condition but not the second, and the Legendre polynomials fill the second but not the first. The Modified Legendre Polynomials constructed by Jorgensen fill these two conditions and are adopted in this work [11]. These polynomials are given by:

$$\mathbb{B} = \widetilde{L}_n(u) = \begin{cases} 1-u & n=0 \\ 1+u & n=1 \\ L_n(u) - L_{n-2}(u) & n \geq 2 \end{cases} \tag{11}$$

where L is the Legendre polynomials in the interval of $u\,]-1\,1[$. Two first terms of Modified Legendre polynomials can be adjusted to impose the continuity and the higher order terms are zero in the extremities and don't have the influence to the continuity.

Although the HO-MoM allows the use of the larger mesh-length than the Classic-MoM, this advantage cannot be exploited for the high-curvature objects with the standard meshing-technique (linear segments). For these objects, we can use the Non Uniform Rational Basis Splines (NURBS) meshing technique. The use of NURBS in electromagnetic modeling are introduced by Spanish scientist [12] and its combinaison with HO-MoM can find in the recent articles [13,14].

To illustrate how HO-MoM + NURBS solve the electromagnetic scattering, we compute the surface currents of an infinite longer cylinder (2D problems). The surface is discretized into 6 segments with the mesh-length $dL = \lambda$, bigger than the standard mesh-length imposed by the classical MoM (figure (4)). As reference, we take the analytical results (exact solution) in form of the Henkel function series [15].

The surface currents for the horizontal and vertical polarization are given in figure (5). We see that the pulse and triangle basis function cannot approach the surface currents with this mesh-length. For the 2nd order, there are the big oscillation of the currents in the extremities of the mesh. Increasing the order of the basis (until 5th order in this example) can approach the exact currents and decreases the oscillation. From these surface currents, we can compute the field every where in the space. More detail results of the accuracy of the HO-MoM are discussed in [16].

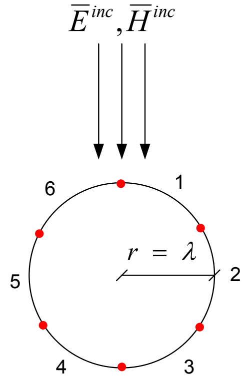

Fig. 4: Scattering from an infinite cylinder (circle)

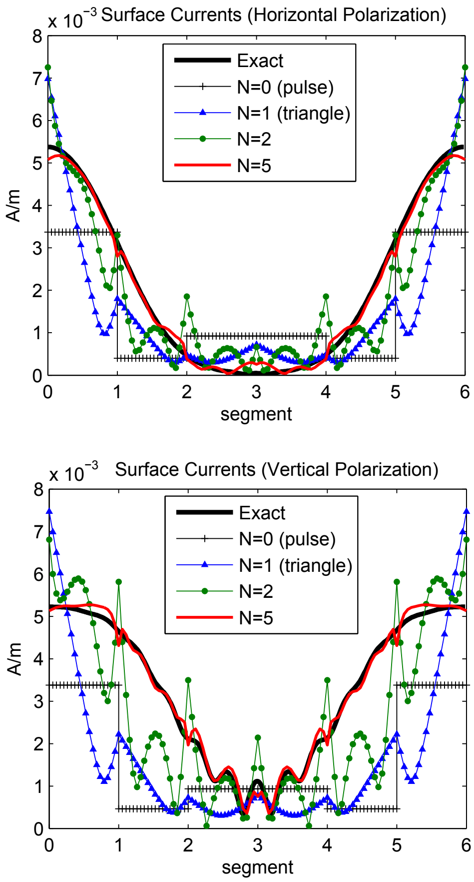

Fig. 5: Surface Currents in horizontal and vertical polarization

4 Simulation results

The dispersion in the crest and the multiple reflexion in the cavity are two mainly factors which influence the sig-

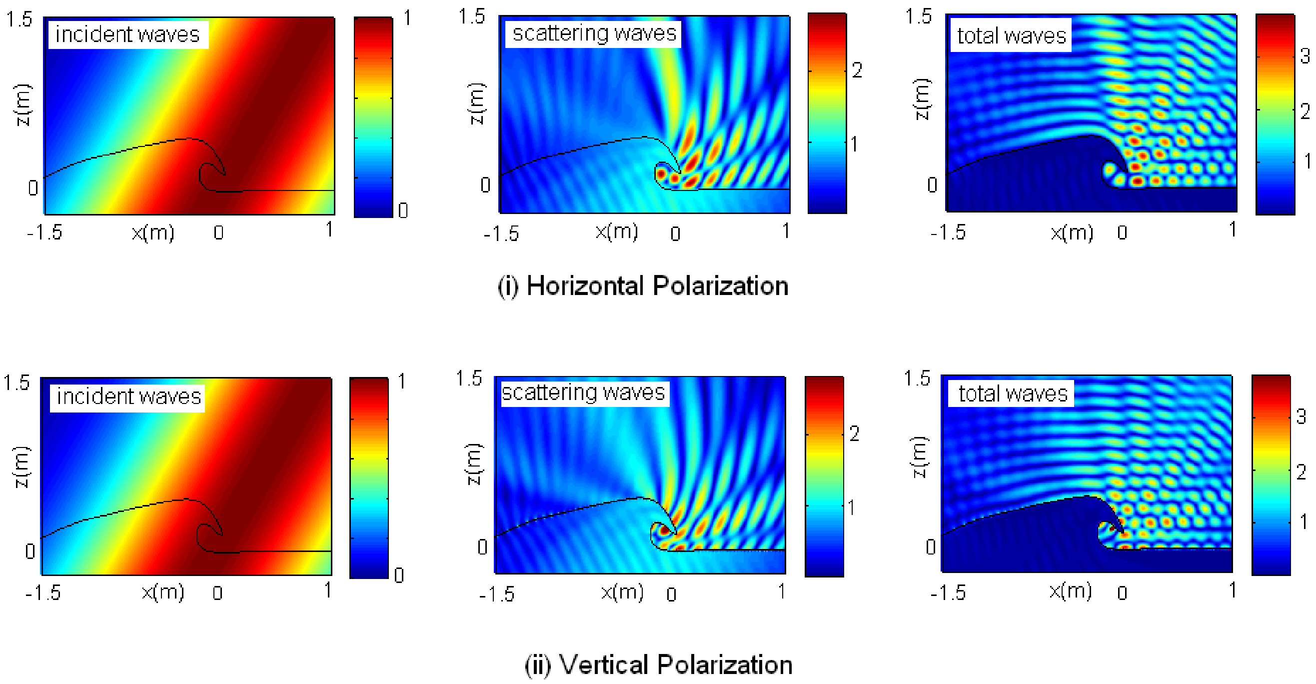

Fig. 6: Electromagnetic field scattering in crest and cavity

nature of the waves scattering from breaking waves. These phenomenas, shown in figure (7) are the reason why of our numerical modeling. The asymptotic methods cannot model accurately these phenomenas. Thus, we use the numerical method (Method or Moments)

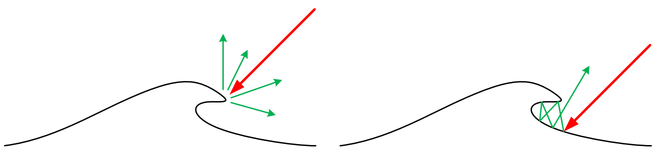

Fig. 7: Scattering frome the crest and cavity of the breaking waves

In the MoM simulation, the breaking waves profiles have to be limited in finite area. Using a pure plane waves as the incident produce the artificial reflection in the edge of the surface. To solve this problem we can use the tapered incident wave [17]. Besides, since the breaking waves have the strong curvature, we use the High-Order Method of Moments combine with NURBS meshing.

The electromagnetic waves scattering in horizontal and vertical polarization for the last profile of (3) are presented in figure (6). In these figure, the cavity resonance phenomenon clearly appears and the relative positions of the node and the anti-node in the different polarizations agrees with the theoretical boundary conditions.

Finally, to obtain the repartition of the energy, we can use the Radar Cross Section (RCS). We see that the most part of the energy are scattered in to the specular direction. However, the crest of the sea wave induce a significant scattering in a large angular domain.

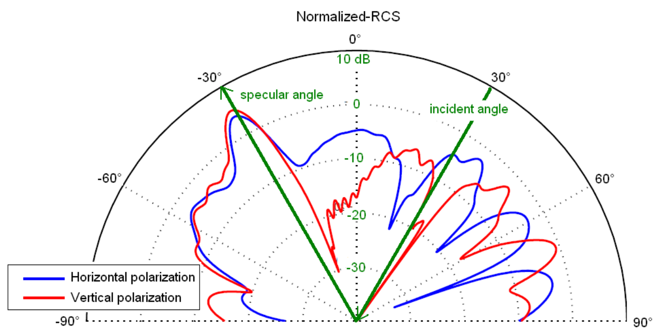

Fig. 8: Radar Cross Section

5 Conclusion

We have shown how a High-Order Method of Moments (HO-MoM) combined with a Non Uniform Rational Basis Spline (NURBS) geometry can compute precisely the electromagnetic wave scattering from breaking sea waves and can evaluate the electromagnetic field where the curvature of the profile is strong (crest for instance). This numerical approach generates numerical simulations of the electromagnetic scattering by a given breaking wave profile (3) for the horizontal and vertical polarizations.

In the present study, the simulations are limited to the 2D problem and only co-polarization can be investigated. In future studies, we will introduce 3D breaking wave profiles to analyze the four components of the scattering matrix (co- and cross-polarization).

References

1. A. Coatanhay, R. Garello, B. Chapron, and F. Ardhuin, "Project MOPS - Marine Opportunity Passive Systems," in *Passive'08*, Hyères, 2008.
2. Q. Ma, Ed., *Advances in numerical simulation of nonlinear water waves.* World Scientific, 2010.
3. P. Wang, Y. Yao, and M. P. Tulin, "An efficient numerical tank for non-linear water waves, based on the multi-subdomain approach with bem," *Int. J. Numer. Methods Fluids*, vol. 20, no. 12, p. 13151336, 1995.
4. Y. M. Scolan, "Some aspects of the flip-through phenomenon: A numerical study based on the desingularized technique," *Journal of Fluids and Structures*, vol. 26, pp. 918–953, 2010.
5. J. C. West, "Ray analysis of low-grazing scattering from a breaking water wave," *Antennas and Propagation, IEEE Transactions on*, vol. 37, no. 6, pp. 2725–2727, 1999.
6. J. C. West and Z. Zhao, "Electromagnetic modeling of multipath scattering from breaking water waves with rough faces," *Antennas and Propagation, IEEE Transactions on*, vol. 40, no. 3, pp. 583–592, 2002.
7. C. Davis and K. Warnick, "Error analysis of 2-D MoM for MFIE/EFIE/CFIE based on the circular cylinder," *Antennas and Propagation, IEEE Transactions on*, vol. 53, pp. 321 – 331, 2005.
8. R. M. Sorensen, *Basic Coastal Engineering.* Springer, 2006.
9. C. J. Galvin, "Breaker type classification," *J. Geophys. Res*, vol. 73, pp. 3651–3659, 1968.
10. M. Sadiku, *Numerical Techniques in Electromagnetics with MATLAB.* CRC Press, 2009.
11. E. Jorgensen, J. L. Volakis, P. Meincke, and O. Breinbjerg, "Higher order hierarchical legendre basis functions for electromagnetic modeling," *Antennas and Propagation, IEEE Transactions on*, vol. 52, pp. 2985–2995, 2004.
12. L. Valle, F. Rivas, and M. F. Catedra, "Combining the moment method with geometrical modelling by NURBS surfaces and Bézier patches," *Antennas and Propagation, IEEE Transactions on*, vol. 42, pp. 373–381, 1994.
13. H. Yuan, N. Wang, and C. Liang, "Combining the higher-order method of moments with geometric modelling by NURBS surfaces," *Antennas and Propagation, IEEE Transactions on*, vol. 57, pp. 3558–3563, 2009.
14. Z. L. Liu and J. Yang, "Analysis of electromagnetic scattering with higher-order moment method and NURBS model," *Progress in Electromagnetic Research*, vol. 96, pp. 83–100, 2009.
15. W. Gibson, *The Method of Moments in Electromagnetics.* Chapman & Hall/CRC, 2008.
16. R. Khairi, A. Coatanhay, and A. Khenchaf, "A 2D high-order method of moments combined with NURBS," Progress in Electromagnetic Research (Submitted).
17. H. Braunisch, Y. Zhang, C. Ao, S. Shih, Y. Yang, K. Ding, J. Kong, and L. Tsang, "Tapered wave with dominant polarization state for all angles of incidence," *Antennas and Propagation, IEEE Transactions on*, vol. 48, no. 7, pp. 1086–1096, July 2000.
18. R. Harrington, *Field Computation by Moments Method.* IEEE Press, 1993.
19. F. Molinet, "Edge-excited rays on convex and concave structures: a review," *Antennas and Propagation Magazine, IEEE*, vol. 47, no. 5, pp. 34 – 46, oct. 2005.
20. ——, "Uniform asymptotic solution for the diffraction by a discontinuity in curvature," *Annales Des Télécommunications*, vol. 50, pp. 523–535, 1995.