Minimum Entropy Approach for Carrier Frequency Recovery

Maciej Pedzisz and Arnaud Coatanhay

Abstract— This paper introduces a new carrier frequency recovery approach. It can be applied before modulation classification and/or demodulation of the M-ary PSK signals. It relies on the entropy of the instantaneous phase probability density function, and uses the fact that it reaches minimum when the receiver is fine-tuned to the unknown carrier frequency. This estimator is applicable to algorithms requiring high accuracy without any a priori knowledge concerning modulation scheme, signal contents (bit-stream), or its timing parameters. Simulation results have proved the robustness of the algorithm: for low Carrier to Noise Ratios (CNR), corresponding variances are proportional and close to Cramér-Rao Lower Bounds (CRLB). For CNR greater than 20 dB, they are constant and limited by the resolution of the algorithm.

Index Terms— Carrier recovery, constellation stabilization, synchronization.

I. INTRODUCTION

IN recent years, many researchers have focused their attention on designing automatic modulation classification N recent years, many researchers have focused their at-(recognition) algorithms. The performance and complexity of these algorithms depend on the number of unknown parameters in the intercepted transmission. One of the most important signal parameters is its carrier frequency, which allows stabilizing signal constellation and recognizing the underlying modulation type.

Most published papers in the field of carrier recovery deal with cases in which some signal parameters are known. In [1], authors obtained their estimator for two different communication scenarios: a TDMA satellite link using standard modulation and burst formats, and a mobile cellular terrestrial radio system with signal and channel characteristics obeying GSM recommendations. In both cases they assumed knowledge of the training bits and symbol timing.

Similar assumptions were made in [2], where perfect symbol synchronization, absence of intersymbol interferences (ISI), and known symbol rate were taken into consideration. Their Maximum Likelihood (ML) Non-Data-Aided (NDA) algorithm performs well for 4QAM signals at moderate or high CNR. However in the 16QAM case, their estimator makes large errors.

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In [3], authors presented the Data-Aided (DA) ML estimator, whose performance is close to the Modified Cramér-Rao Bound (MCRB) for low Signal to Noise Ratios (SNR), and known data sequence. They compare this method to standard ML algorithms proposed by Kay [4] and Fitz [5] (used as references).

Low estimator variance was achieved in a two-step approach proposed in paper [6], where the estimators of unknown channel characteristics and frequency offset were based on the information provided by the second or fourth-order cyclostationary (CS) statistics.

The adaptive filter theory was used in an algorithm proposed in [7], where a new data modulation removal method, as well as an adaptive Recursive Least Squares (RLS) filter were applied to improve the performance of the algorithm. It was assumed that symbol timing was known to the receiver, and the output signal samples were taken from the output of a matched Pulse Shaping Filter (PSF) (one sample per symbol).

In [8], authors introduced a family of blind feedforward nonlinear estimators for joint estimation of the carrier phase, the frequency offset and the Doppler rate for burst-mode PSK transmissions. The asymptotic performances of these estimators were established in a closed-form expression and compared with the CRLB for an unmodulated carrier.

In this paper we propose a NDA asynchronous approach, i.e. neither preamble sequence nor prior knowledge about the datastream and the timings are available. We assumed a MPSK signal without any additional information concerning the number of states, the initial carrier phase, or the transmission baud rate. In Section II, the signal model is presented and some assumptions are made. Section III presents the theoretical basis related to an instantaneous phase probability density function, and Section IV deals with its entropy. The description of the algorithm is given in Section V, and the corresponding results are illustrated in Section VI. Final conclusions are presented in Section VII.

II. SIGNAL MODEL

Let's assume that the received complex signal can be expressed as a sum of two uncorrelated components

$$
r(n) = Ax(n)e^{j(\omega_c nT_s + \Theta_c)} + z(n), \quad 0 \le n < N,\tag{1}
$$

where N is a number of samples, $x(n)$ is a signal complex envelope, A is a carrier amplitude, ω_c is a carrier frequency, Θ_c is a carrier phase, T_s is a sampling interval, and $z(n)$

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corresponds to a complex, zero mean Additive White Gaussian Noise (AWGN).

In the M-ary PSK case, the complex envelope can be expressed as

$$
x_{mpsk}(n) = \sum_{k=0}^{K-1} e^{j\varphi_k} u(nT_s - kT),
$$
 (2)

where K is a number of observed symbols, T is a symbol duration, φ_k represents constellation of the signal $\varphi_k \in$ duration, φ_k represents constellation or the signal $\varphi_k \in$
 $\left\{ \frac{2\pi}{M}(m + \frac{1}{2}) - \pi, m = 0, 1, ..., M - 1 \right\}$, and $u(n)$ is a pulse shaping function.

If in equation (1) the signal complex envelope $x(n)$ is a constant value, then it describes a Carrier Wave (CW) signal

$$
r_{cw}(n) = Ae^{j(\omega_c nT_s + \Theta_c)} + z(n). \tag{3}
$$

It is assumed also that all modulation states φ_k are equiprobable and that the pulse shaping function $u(n)$ is rectangular.

III. INSTANTANEOUS PHASE PROBABILITY DENSITY **FUNCTION**

Let's focus on the CW signal. If it is expressed as in (3) , and if its carrier frequency ω_c and its initial phase Θ_c are exactly known, after complex mixing with a local generator $e^{-j(\omega_{LO}nT_s+\Theta_{LO})}$ ($\omega_{LO} = \omega_c, \Theta_{LO} = \Theta_c$) we obtain a complex baseband signal, whose probability density function (PDF) of its instantaneous phase ψ (IP) may be written as [9]

$$
p_{cw}(\psi) = \frac{e^{-\gamma}}{2\pi} + \frac{e^{-\gamma}}{2} \sqrt{\frac{\gamma}{\pi}} \cos(\psi) e^{\gamma \cos^2(\psi)}
$$

$$
\{1 + \text{erf}[\sqrt{\gamma} \cos(\psi)]\}, \quad \psi \in [-\pi, \pi[, \qquad (4)
$$

where $\gamma = \frac{A^2}{2\sigma^2}$ $\frac{A^2}{2\sigma_z^2}$ = 10^{CNR/10}, σ_z^2 is a noise variance, and $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}$ $\frac{2\sigma_z^2}{\int x}$ $\int_0^{\tilde{x}} e^{-t^2} dt$ is an error function.

Extending the domain of ψ to infinity by duplicating $[-\pi, \pi]$ range, we obtain a periodic function which meets Dirichlet conditions, and can be written as an even term Fourier series ([10], [11])

$$
p_{cw}(\psi) = \frac{1}{2\pi} \left[1 + \sum_{l=1}^{\infty} b_l \cos(l\psi) \right],
$$
 (5)

with the Fourier series coefficients

$$
b_l = \sqrt{\pi \gamma} e^{-\frac{\gamma}{2}} \left[I_{\frac{l-1}{2}} \left(\frac{\gamma}{2} \right) + I_{\frac{l+1}{2}} \left(\frac{\gamma}{2} \right) \right],\tag{6}
$$

where $I_l(x)$ is the modified Bessel function of order l.

Taking into consideration the symmetry of the MPSK constellations, the IP PDF can be expressed as

$$
p_{mpsk}(\psi) = \frac{1}{M} \sum_{k=0}^{M-1} p_{cw} \left[\psi + \frac{2\pi (k + \frac{1}{2})}{M} - \pi \right].
$$
 (7)

It should be noted that this equation holds when the phase of the local oscillator is equal to the initial phase of the signal. When they are different, the corresponding IP PDF is shifted on the ψ axis, and the expression in brackets must be rewritten as

$$
\psi + \frac{2\pi (k + \frac{1}{2})}{M} - \pi + \Theta_c - \Theta_{LO}.
$$
 (8)

Let us analyze the IP PDF of N samples of the CW signal with unknown frequency ω_c . The error between the true carrier frequency and the estimated one ($\Delta_{\omega} = \omega_c - \omega_{LO}$) corresponds to the slope of the unwrapped IP and can be described as a phase shift (or an angle of the constellation rotation in a scatter plot) as follows:

$$
\alpha = \Delta_{\omega}(N-1)T_s. \tag{9}
$$

This phase shift is responsible for changing the resulting IP PDF, which can be modeled as a convolution of the $p_{cw}(\psi)$ with a rectangular function written as

$$
\Pi(\psi) = \begin{cases} 1/\alpha & \text{for } \psi \in [-\alpha/2, \alpha/2], \\ 0 & \text{otherwise}, \end{cases}
$$
(10)

and in terms of the Fourier series expansion:

$$
\Pi(\psi) = \frac{1}{2\pi} \left[1 + \sum_{l=1}^{\infty} 2 \frac{\sin\left(\frac{l\alpha}{2}\right)}{\frac{l\alpha}{2}} \cos\left(l\psi\right) \right].
$$
 (11)

Using the convolution theory, the resulting IP PDF can be expressed as follows:

$$
p_{cw}^{\Delta_{\omega}}(\psi) = \int_{-\infty}^{+\infty} \Pi(\psi - \phi) p_{cw}(\phi) \, d\phi = \mathcal{F}^i \left\{ \mathcal{F}_{cw}^f(\nu) \cdot \mathcal{F}_{\Pi}^f(\nu) \right\},\tag{12}
$$

where \mathcal{F}^f and \mathcal{F}^i are adequately forward and inverse Fourier Transform operators expressed as

$$
F(\nu) \stackrel{\mathcal{F}^f}{=} \int_{-\infty}^{+\infty} f(\psi) e^{-j\nu\psi} d\psi; \quad f(\psi) \stackrel{\mathcal{F}^i}{=} \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\nu) e^{j\nu\psi} d\nu.
$$
\n(13)

Using now the relation

$$
\cos(k\psi) \stackrel{\mathcal{F}^f}{\Leftrightarrow} \frac{1}{2} [\delta(\nu - k) + \delta(\nu + k)],\tag{14}
$$

and equations (12) and (13), the final IP PDF can be written as .
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$$
p_{cw}^{\Delta_{\omega}}(\psi) = \frac{1}{2\pi} \left[1 + \sum_{l=1}^{\infty} d_l \cos(l\psi) \right],
$$
 (15)

with the coefficients d_l

$$
d_l = \frac{\sqrt{\pi \gamma} \sin\left(\frac{l\alpha}{2}\right)}{\frac{l\alpha}{2}} e^{-\frac{\gamma}{2}} \left[I_{\frac{l-1}{2}}\left(\frac{\gamma}{2}\right) + I_{\frac{l+1}{2}}\left(\frac{\gamma}{2}\right) \right].
$$
 (16)

To obtain a general solution for the MPSK signals, one can exchange the $p_{cw}(\psi)$ in equation (7) with the $p_{cw}^{\Delta_{\omega}}(\psi)$ expressed as in (15)

$$
p_{mpsk}^{\Delta_{\omega}}(\psi) = \frac{1}{M} \sum_{k=0}^{M-1} p_{cw}^{\Delta_{\omega}} \left[\psi + \frac{2\pi (k + \frac{1}{2})}{M} - \pi \right].
$$
 (17)

It is worth noting that the equations (15) and (17) hold for $\alpha \in [0, 2\pi]$. When α exceeds this range, both of them can be approximated by the uniform distribution $1/2\pi$.

IV. ENTROPY OF THE INSTANTANEOUS PHASE PROBABILITY DENSITY FUNCTION

The entropy of a random variable is a quantitative measure of the randomness of the corresponding experiment. It can be

Fig. 1. Entropy of the IP PDF with respect to α for CW, BPSK, QPSK, and 8PSK signals at CNR = 15 dB.

expressed in terms of a PDF $p(x)$ [12] as

$$
H_x = -\int_{-\infty}^{+\infty} p(x) \ln [p(x)] \, dx. \tag{18}
$$

Applying this idea to the IP PDF of a MPSK signal, we obtain

$$
H_{mpsk}^{\Delta_{\omega}} = -\int\limits_{-\pi}^{+\pi} p_{mpsk}^{\Delta_{\omega}}(\psi) \ln \left[p_{mpsk}^{\Delta_{\omega}}(\psi) \right] d\psi. \tag{19}
$$

One should note that applying the equations (17) and (15) in the expression (19), imposes that the analytical solution is almost impossible to find (logarithm of the infinite sum in the equation 15).

To point out the influence of the α value (frequency error), numerical integration was conducted and the results are presented in Fig. 1. The graphs show that there is an upper limit for the entropy. This corresponds to situations in which the frequency error is big enough that constellation characteristic states can not be seen, and is equivalent to the following conditions: $\alpha \geq 2\pi$ for CW, $\alpha \geq \pi$ for BPSK, $\alpha \geq \pi/2$ for QPSK, and $\alpha \ge \pi/4$ for 8PSK. These particular angles correspond to the maximal frequency error

$$
\Delta_{\omega}^{\max} = \frac{2\pi}{M(N-1)T_s} \tag{20}
$$

above which the IP PDF can be approximated as the uniform distribution $1/2\pi$ with the corresponding entropy

$$
H_{\text{max}} = \ln(2\pi) \approx 1.838. \tag{21}
$$

On the other hand, the entropy reaches its global minimum for all the signals when $\alpha = 0$ (frequency error $\Delta_{\omega} = 0$). To prove this, it is necessary to simplify the equation (4) by approximating it via the Gaussian distribution [13]

$$
p_{cw}(\psi) \simeq N\left(0, 1/\sqrt{2\gamma}\right). \tag{22}
$$

Fig. 2. Entropy of the IP PDF with respect to CNR for CW, BPSK, QPSK, and 8PSK signals when $\alpha = 0$.

Convolving this distribution with the rectangular one, we obtain the approximated distribution expressed as

$$
p_{cw}^{\Delta_{\omega}}(\psi) = \frac{1}{2\alpha} \left[\text{erf}\left(\sqrt{\gamma}\left(\psi + \frac{\alpha}{2}\right)\right) - \text{erf}\left(\sqrt{\gamma}\left(\psi - \frac{\alpha}{2}\right)\right) \right] \tag{23}
$$

and the corresponding entropy can be written as a Taylor series expansion around $\alpha = 0$ point

$$
H_{cw}^{\Delta_{\omega}}\Big|_{\alpha \to 0} =
$$

$$
\ln\left(\sqrt{\frac{\pi e}{\gamma}}\right) + \frac{\alpha^2 \gamma}{12} - \frac{\alpha^4 \gamma^2}{144} + \frac{\alpha^6 \gamma^3}{1296} - \frac{31\alpha^8 \gamma^4}{259200} + O(10)(24)
$$

From this equation and Fig. 1, it follows that around the point $\alpha = 0$ the entropy has a global minimum corresponding to the case $\Delta_{\omega} = 0$. It can be expressed by the entropy of a Gaussian distribution¹ [12] as

$$
H_{\min} = \ln\left(\sqrt{\frac{\pi e}{\gamma}}\right) \tag{25}
$$

To provide insight into how the entropy changes as a function of CNR, numerical integration was conducted and the corresponding results are presented in Fig. 2. Based on these graphs, it is clear that there are some theoretical limits for carrier frequency detectability using this method. For example, the CNR must be greater than 10 dB for 8PSK, 5 dB for QPSK, and -5 dB for BPSK to properly identify the minimum of the entropy.

It is worth noting that the entropy depends only on the shape of the IP PDF, and the amplitude of the received signal, as well as the initial carrier and local oscillator phases do not change its value.

V. ALGORITHM DESCRIPTION

The algorithm can be decomposed into two main parts: raw estimation, and fine-tuning. The aim of raw estimation is to provide an approximate value of the carrier frequency, around

¹There is no convolution with the rectangular function.

Fig. 3. Performances of the proposed algorithm.

which the fine-tuning part can search for the minimum value of the IP PDF entropy.

At the beginning, the algorithm estimates the Power Spectral Density (PSD) of a signal. It uses the Welch [14] modified periodogram method because of its ability to control the bias and the variance. Next, an experimentally chosen threshold is applied to the PSD to extract only the meaningful part of the signal spectrum. Finally, the mean value of the extracted part is calculated. This value is used as the raw carrier frequency estimator around which the Frequency Raster is constructed (the surroundings of the mean frequency).

The fine-tuning part is implemented as a downconversion with the frequencies chosen from the Frequency Raster. Resulting baseband signal samples are used to extract the IP PDF and the corresponding entropy. Finally, the minimum searching algorithm is applied to find the minimum value among all the entropies.

One must pay attention to two topics in particular: the minimum searching algorithm and the computational complexity of the algorithm. The fact that we do not assume any a priori knowledge concerning the location of the true carrier frequency makes it necessary to apply the raw estimation part. The quality of this estimator depends on the number of points used during PSD estimation (number of the FFT points and the overlapping factor). To make the algorithm more robust to the errors arisen due to the PSD estimation, one must create the Frequency Raster of adequate size (large neighborhood and large number of frequencies). This implies that there is a sharp peak in the Frequency Raster (corresponding to the minimal entropy), and that the rest of the Raster is almost constant. Such a minimum can not be found using classical gradient techniques, and as a consequence, one must apply the linear search algorithm instead of more efficient ones. To sum up, the errors due to raw estimation part increase the size of the Frequency Raster, which means that the application of any gradient techniques is impossible, and the computational complexity is important. The methods of reducing both factors will be addressed elsewhere.

VI. SIMULATION RESULTS

The performance of the proposed carrier frequency estimation algorithm was assessed by computer simulations with the following assumptions: CW, BPSK, QPSK and 8PSK modulation types; 4096 complex signal samples for each trial; 1000 trials for each signal, source signals were modeled as uniformly distributed on all the constellation states; baud rate was varying from 100 to 1000 Bd with the step of 100 Bd; additive noise was modeled as Gaussian; CNR was varying from 0 to 30 dB with the step of 5 dB; sampling frequency F_s was equal 31.7365 kHz; and the receiver filter bandwidth was 3.4 kHz.

The simulation results have proved that the proposed estimator is unbiased and independent of the baud rate for all signals. Variances of the normalized frequency error

$$
Var(\delta_f) = Var(\Delta_f/F_s)
$$

= $Var((\hat{f}_c - f_c)/F_s) = Var((f_{LO} - f_c)/F_s)$

for all signals and different CNR values are presented in Fig. 3 by thin lines.

To grasp the performance level of the proposed estimators, it is necessary to compare the results with the theoretical CRLBs. Following the Rife [15], it can be shown that for the CW signal of length N samples in AWGN, CRLB may be expressed as

$$
CRLB_{CW} = \frac{6\sigma_z^2}{A^2 N (N^2 - 1)(2\pi)^2},
$$
 (26)

and using the relation $A^2/\sigma_z^2 = 2 \cdot 10^{\text{CNR}/10}$, as

$$
CRLB_{CW} = \frac{3 \cdot 10^{-\text{CNR}/10}}{N(N^2 - 1)(2\pi)^2}.
$$
 (27)

Unfortunately, this relation holds only for the simplest cases. When MPSK signals are taken into consideration, no general closed-form expression is available. One possible solution for this problem is to use the so-called modified CRB (MCRB) as in [16], or the low-SNR asymptotic CRB (ACRB) as in [17]. The other solution is to use numerical integration as in [18] to find the bounds for MPSK signals. Using the second approach, it can be shown that CRLB_M (for MPSK signals) is asymptotically equal to CRLB_{CW} for large SNR. The corresponding curves are represented in Fig. 3 by bold lines.

It is worth noting that the variances of the estimators are approximately constant for $\text{CNR} > 20$ dB and are limited by the resolution of the Frequency Raster. When CNR < 20 dB, the variances are approximately proportional to the corresponding CRLBs.

VII. CONCLUSION

The proposed algorithm can be applied for all types of transmissions – it estimates the mean frequency of the signal spectrum (raw estimation). In the case of a linearly, digitally modulated signal, as MPSK, it estimates the corresponding carrier frequency. It is independent of the signal level, initial carrier phase and timings.

The method can be implemented as an "one-time estimator" (using the entire algorithm), or as a "tracking estimator" (using the fine-tuning part and the properties around $\alpha = 0$ point).

The simulation results have proved the robustness of the algorithm: for low CNR corresponding variances are proportional and close to CRLB; for CNR greater than 20 dB they are constant, independent of the type of the signal and limited by the resolution of the Frequency Raster (which is sufficient in most practical implementations).

Further work will be conducted to improve the quality of the raw estimator (e.g. using a Yule-Walker AR modeling to estimate the PSD), the fine-tuning part (e.g. two or more Frequency Rasters), as well as decrease the computational overhead (e.g. exchanging the Shannon entropy with the Renyi entropy [19]).

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